Resilient Recovery on Stochastic Chaotic Supply Chain Networks Using Pinning Control

Recuperación resiliente de redes caóticas estocásticas de cadenas de suministro mediante control tipo PIN

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Abstract: In this paper, a novel pinning control scheme is proposed to achieve synchronization in a supply chain network, which is regarded as a dynamic system consisting of member companies with dynamic behaviors and different enterprise relations. This study considers that the network is influenced by two different noises and/or uncertainties modeled through stochastic variables and the internal dynamics in the companies are chaotic. The main objective of the proposed scheme is to achieve the resilient recovery of a disrupted supply chain network. Moreover, we illustrate the effectiveness of the approach by means of a real case supply chain network.

Keywords: Resilient recovery, supply chain network, stochastic complex network, pinning control, chaotic system.

Resumen: En este artículo se propone un novedoso esquema de control tipo PIN para alcanzar la sincronización en una red de cadena de suministro, concebida como un sistema dinámico compuesto por empresas miembro con comportamientos dinámicos y diversas relaciones empresariales. En este estudio se considera que la red está sujeta a dos tipos distintos de ruido y/o incertidumbre, modelados mediante variables estocásticas, y que la dinámica interna de las empresas es caótica. El objetivo principal del esquema propuesto es lograr la recuperación resiliente de una red de cadena de suministro afectada por disrupciones. Asimismo, se ilustra la eficacia del enfoque mediante un caso real de red de cadena de suministro.

Palabras clave: Recuperación resiliente, red de cadena de suministro, red compleja estocástica, control tipo PIN, sistema caótico.

1. INTRODUCTION

Complex networks have allowed us to describe many systems ubiquitous in nature and society, such as biomolecular networks, neural networks, the spread of infectious diseases, social relations, World Wide Web, electrical power grids, and so on [1]–[3]. A common interesting behavior that it is presented in these networks is the network synchronization [4]–[6]. Different approaches have been proposed to study the synchronization of complex dynamical networks without control [7]–[9].

A common complex network can be the supply chain process, where it is composed of various types of stakeholders named customers, enterprises, suppliers and so on which have their own objectives that usually are not aligned with the goals of the company [10]. The high complexity of the Supply Chain Processes is given by the several interconnections of nodes, the dynamism of events that make it harder to predict future behavior. When a disruptive event occurs in a supply chain network and given the complexity of the network structure there is a high need to develop strategies that allow the system to be restored to the normal operation [11]. Such disruptions are caused by unpredictable events like epidemics, disasters, and/or terrorist attacks which increase the economic and operational losses [12]. Some results have been proposed to supply chain networks in the framework of complex dynamical networks [13], [14]. Geng and Xiao propose a resilient recovery method based on synchronization for a deterministic supply network [13]; the same authors extend this work to analyze synchronization-based resilient recovery measures with memory and control, identifying backbones [14]. Pinning control is a technique used toward a desired collective behavior when a network cannot do it by itself. This technique is very convenient for huge-size systems, i.e., with a great number of nodes (like the supply chain networks introduced before), given that the approach of this technique consists of applying appropriate manner control signals to a small fraction of nodes in the network [15]–[17]. The main objective of this technique is to find a set number and suitable location of nodes to be pinned in order to achieve the control goal effectively.

This type of approach is based on node dynamics and network topology as well as the network evolution. Accordingly, since the first proposed work in the design of pinning controllers [18], [19], many efforts research have been developed to analyze this topic, including stability conditions, complete synchronization, cluster synchronization, selective strategies for the pinned nodes, controllability properties, and design of different control methodologies, pursuing to extend the classical control analysis to complex networks [20]–[24].

As can be noted, many advances in pinning control of complex networked systems have been made. Although, the majority are framed in deterministic systems with noise-free. An area with great potential which presents a great challenge is the stochastic complex networks. These systems have received much attention due to their wide applicability in many areas of engineering, for systems, which undergo wind forces, earthquake excitation, electrical current fluctuating due to thermal noise, sunspots in economic models, and any other random variables [25], which allows obtaining model more reliable.

There have been different research in the framework of stochastic complex networks [26]–[31]. The stochastic synchronization problem is addressed in [26] and [27] for the case delayed complex dynamical networks with nonidentical nodes. The impulsive synchronization is analyzed [28] for networks with time delays and switching topologies. In [29] sufficient conditions for global synchronization are derived. Three different types of noise diffusion processes affecting the complex network individually are analyzed in [30], conditions for pinning controllability are derived, assuming that the control law has the same form as the coupling function. Using the master stability function approach, locally necessary and sufficient conditions for synchronization are provided [31].

With the above motivations, this paper addresses the global stochastic synchronization problem via pinning control law applied to stochastic supply chain chaotic processes to guarantee resiliency recovery. The effects caused by two different types of noise diffusion processes are taken into account. The complete synthesis of the pinning controller is provided, where a methodology for the selection of the controlled nodes, the minimal coupling strength required, and the bound of the pinning control law, to achieve the control goal is developed.

A European supply chain network case is used to illustrate the effectiveness of the proposed scheme, where the flows between the stakeholders are represented by links of communication with noise, and the business dynamics are considered chaotic and stochastic.

Notations: I_n stands for the $n \times n$ identity matrix. $A > 0$ ($A < 0$) means A is positive (negative) definite. A^{T} is the transpose of the matrix A. $\|\cdot\|$ stands for the operator norm. $\mathrm{Tr}\{A\}$ is the trace of a square matrix $A = (a_{ij})_{d\times d}$, i.e. $\mathrm{Tr}\{A\} = \sum_{i=1}^{d} a_{ii}$. $\lambda_{\min}(\cdot)$ ($\lambda_{\max}(\cdot)$) represents the smallest (largest) eigenvalue of the corresponding matrix. $f : A \to B$ is the mapping f from A to B. $A \otimes B$ represents their Kronecker product with the following properties:

1) $(A \otimes B)^T = A^T \otimes B^T$
2) $(\alpha A) \otimes B = A \otimes (\alpha B)$
3) $(A + B) \otimes C = A \otimes C + B \otimes C$
4) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$

2. FUNDAMENTALS

2.1. Stochastic chaotic supply chain network model

A mathematical model for stochastic chaotic supply chain networks is proposed by using the framework of complex networks [15]. Consider a general network consisting of N diffusively coupled identical nodes coupled on an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_i\}$ is the set of companies nodes involved in the network (Suppliers, factories, distributors); $\mathcal{E} = \{e_{ij}\}$ is the set of relations between supply and demand, representing the different flows between companies (nodes). The network dynamics is described by the following set of nonlinear stochastic differential equations of Itô type:

$$\mathrm{d}x_i = (f(x_i) - c\sum_{j=1}^{N} L_{ij}h(x_j) + u_i)\mathrm{d}t$$

$$+ \sigma_m g(x_i)\mathrm{d}b_m + \sigma_c \sum_{j=1}^{N} L_{ij}^{*} h(x_j)\mathrm{d}b_c$$

$$i = 1, 2, \ldots, N, \qquad (1)$$

where $x_i = [x_{i1}, \ldots, x_{in}]^{\mathrm{T}} \in \mathrm{R}^n$ is the state vector of node i; $f, h : \mathrm{R}^n \to \mathrm{R}^n$ are the twice differentiable vector fields describing the deterministic individual dynamics and the output coupling function, respectively; $L = [l_{ij}] \in \mathrm{R}^{N\times N}$ is the Laplacian matrix describing the network configuration, when there is a business relationship between company node i and j, $l_{ij} = 1$, otherwise $l_{ij} = 0$, ; $c > 0$ is the coupling strength among neighboring nodes; u_i is the control input. $\sigma_m g(x_i)\mathrm{d}b_m$ represents modeling uncertainty or noisy company dynamics, with b_m being a one-dimensional Wiener process, σ_m modulating its variance, and $g : \mathrm{R}^n \to \mathrm{R}^n$ being the nonlinear diffusion function; final the term given

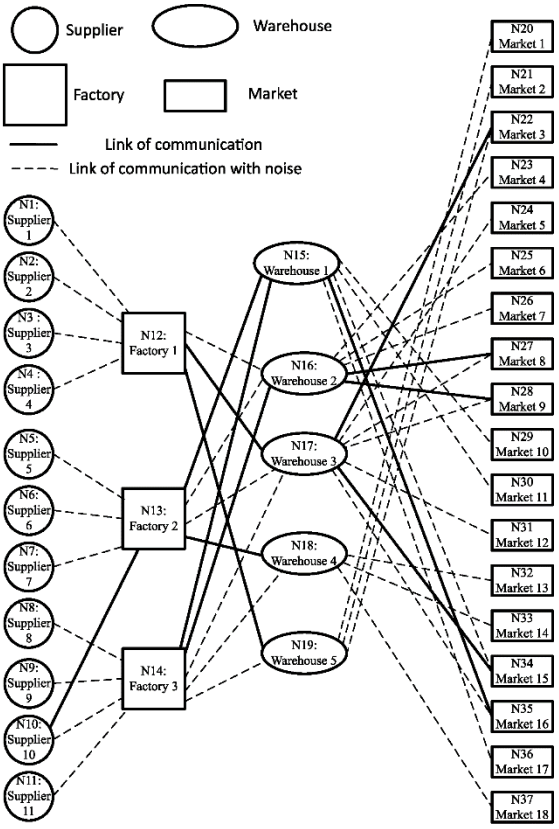


Fig. 1 *Topology of the European supply chain network, including the link communication with noise.*
Source: *Authors.*

$\sigma_c \sum_{j=1}^{N} l_{ij}h(x_j)\mathrm{d}b_c$ represents flows uncertainties or noise presenting by flows, with b_c being a one-dimensional Wiener process, independent of b_m, and σ_c modulating its intensity.

The topology of the supply chain network considered for the present study, as presented in Fig. 1, describes a classic supply chain in which raw materials and parts are acquired from a set of suppliers which send these materials to production plants nodes represented by Factories. Produced goods are then sent to distribution centers where picking, packing, distribution, and transportation tasks are performed to prepare and transport orders to locations of customers at markets.

2.2. Node dynamic model

Three different business dynamic processes are modeled at each company. In the case of the suppliers, each company has a distribution capacity in terms of the number of units they are able to ship to production plant locations, a given geographical location determining the characteristics of travel times and costs, which in turn affect lead times for

distribution, and the current commercial contracts linking suppliers to given distribution plants. The number and connections to factory nodes are fixed. These attributes have an impact on system parameters such as level of service, considered in this study as dissatisfaction of ordering in the previous period, the information distortion degree, and safety stock. Increased lead times could lead to reduced levels of service, bullwhip effects caused by information distortion and delays could lead to increased safety stocks throughout the entire network, increasing costs for instance.

For each type of node in the network, logistics management planning conditions exist determining the order quantities each node places, the inventory levels set and the number of units shipped. Production plants also have a fixed capacity in terms of the number of units they are able to produce and ship, a given location, and a connection to distribution centers. Demand variability can affect production schedules, which in turn have an impact on the level of service and inventory management policy costs. The capacity of distribution operations is also described by the amounts and frequencies each node is able to provide. Different types of logistics operations suppliers can be used at this stage, which also introduces an additional level of noise in lead times, costs, and level of service.

In general, the state of each company (node), x_i, in a supply network are: demand amount for ordering (x_{i1}), supply amount for distributing (x_{i2}), and inventory for production (x_{i3}), i.e., $x_i = [x_{i1}, x_{i2}, x_{i3}]$. The system parameters are: the dissatisfaction of ordering (d_{SC}) in previous period, the information distortion degree (r_{SC}), and the coefficients of safety stock (c_{SC}). The node dynamics, $f(x_i(t), t)$, can be formulated as in [13], [32], where the Lorenz system [33] is used to represent the business dynamical behavior in each company:

$$f(x_i) = \begin{bmatrix} d_{\mathrm{SC}}(x_{i2} - x_{i1}) \\ r_{\mathrm{SC}}x_{i1} - x_{i2} - x_{i1}x_{i3} \\ x_{i1}x_{i2} - c_{\mathrm{SC}}x_{i3} \end{bmatrix}.$$

The systems parameters are selected as $a_{\mathrm{SC}} = 10$, $b_{\mathrm{SC}} = 28$, and $c_{\mathrm{SC}} = 8/3$, to ensure the chaotic behavior of supply chains.

2.3. Network control goal

2.3.1. Network Disruption: The disruption in the supply chain network considered for this study can take different forms. One source of disruption could be parameter distribution shifts due to market changes. Another source of disruption can be node's capacity reductions due to external events such as natural disasters, strikes, political reasons blocking international trade, terrorist attacks, or global disruptive events such as the current Covid-19 pandemic. Finally, network topology changes in terms of the reduction or increase in the number of nodes or connection links in the network can also be a source of disruption. These events can be caused either by the above-mentioned external events, financial crises, or others. They could take the form of certain high-capacity nodes being replaced by several others with lower capacity, logistic operators managing a different subset of links, or changes in commercial agreements between partners. Planning operations and distribution management processes determine the number of units each node orders from prior levels, the internal inventory management policies, and the amounts to ship through each distribution channel.

These operations rely on demand forecasts, which are less accurate when dealing with stochastic demands. They also rely on parameters such as travel times, lead times, costs which not only are stochastic in these types of complex networks but when affected by the above-mentioned problems have a disruptive impact on the network.

2.3.2. Network Resilient Recovery: Resilient recovery of the network is represented as a return to the desired level of performance or scheduled state after a disruption. We can relate this property with the synchronization of the whole network as follows.

Definition 1: Resilient recovery is synchronization process between network and scheduled network.

Therefore, the control goal of $u_i(t)$ is defined as to force (1) to synchronize a reference trajectory of an isolated node, the pinner, $x_r = x_r(t)$, whose dynamics are given by

$$\mathrm{d}x_r = f(x_r)\mathrm{d}t + \sigma_m g(x_r)\mathrm{d}b_m \qquad (2)$$

Consequently, we define the synchronization error as

$$e = [e_1{}^{\mathrm{T}}, \ldots, e_N{}^{\mathrm{T}}]^{\mathrm{T}},$$

where $e_i = x_i - x_r$. The synchronization error dynamics can be defined for each node $i \in \mathcal{V}$, combining (1) and (2), as follows

$$de_i = \left(f(x_i) - f(x_r) - c\sum_{j=1}^{N} L_{ij}h(x_j) + u_i \right) dt + \sigma_m\big(g(x_i) - g(x_r)\big)db_m + \sigma_c \sum_{j=1}^{N} L_{ij}^{*}h(x_j)\, db_c \quad (3)$$

The above control goal can be achieved by applying a minimal quantity of feedback control actions in the network nodes, this is known as pinning control technique [17], [18], which is given by

$$u_i = -p_i\psi e_i, \quad u_i \in \mathrm{R}^n \quad (4)$$

where $p_i = 1$ for $i \in P \subseteq V$, while it is zero otherwise and $\psi > 0$ is a positive constant, which is a control design parameter. Without loss of generality, assume that the first Q nodes are pinned with $Q \leq N$. Then, we denote $P = (p_i\psi) \in \mathrm{R}^{N\times N}$ as a diagonal matrix with Q elements ψ, and its remaining N - Q elements are all zero.

Definition 2: The controlled network (1)-(4) is mean-square pinning controlled onto $x_r(t)$ if there is a constant ψ and a set of pinned nodes $\mathcal{P}$ for which the trivial solution $e(t) = 0$ of the synchronization error dynamics (3) is globally exponentially mean-square stable [34], i.e., if there exist α1, α2 > 0 such that, for all e0 ∈ ℝn, t ∈ ℝ≥0,

$$E[\|e(t)\|^2] \leq \alpha_1 \|e_0\|^2 exp(-\alpha_2 t)$$

Definition 3: [34, Theorem 6.2, pag 32] Consider a positive definite, radially unbounded, twice continuously differentiable function V(e). The operator $\mathcal{L}$ associated to the stochastic Itô equation (3) is defined as:

$$\mathcal{L}V(e) = \frac{\partial V(e)}{\partial e}\phi(e) + \frac{1}{2}\mathrm{Tr}\left\{\eta_1^{\mathrm{T}}(e)\frac{\partial^2 V(e)}{\partial e^2}\eta_1(e)\right\} + \frac{1}{2}\mathrm{Tr}\left\{\eta_2^{\mathrm{T}}(e)\frac{\partial^2 V(e)}{\partial e^2}\eta_2(e)\right\},$$

where

$$\varphi(e) = f(x_i) - f(x_r) - c\sum_{j=1}^{N} L_{ij}h(x_j) + u_i,$$

$$\eta_1(e) = \sigma_m\big(g(x_i) - g(x_r)\big), \text{ and}$$

$$\eta_2(e) = \sigma_c \sum_{j=1}^{N} L_{ij}^{*}h(x_j).$$

Lemma 1: [35, Theorem 5.11] If there exist a positive definite, differentiable function V(e), and three positive scalars β1, β2, β3 such that:

$$\beta_1\|e\|^2 \leq V(e) \leq \beta_2\|e\|^2 \quad (5a)$$

$$\mathcal{L}V(e) \leq -\beta_3\|e\|^2 \quad (5b)$$

then the trivial solution of system (3) is mean-square stable in probability.

Definition 4: The controlled network (1)-(4) satisfies the resilient recovery if the trivial solution e(t) = 0 of system (3) is globally exponentially mean-square stable.

3. PINNING CONTROL DESIGN

Theorem 1: [36] If

(i) for the controlled complex network (1)-(4), there is a positive constant $\theta_f > 0$ such that $(z - y)^T(f(z) - f(y)) \leq \theta_f(z - y)^T(z - y), \forall z, y \in \mathbb{R}^n$;
(ii) function h is globally Lipschitz with constant θ_h and, for all $y, z \in \mathbb{R}^n, t \in \mathbb{R}$, there exist $0 < \theta_l \leq \theta_h$, such that $(z - y)^T(h(z) - h(y)) \geq \theta_l(z - y)^T(z - y)$;
(iii) function g is globally Lipschitz with constant θ_g;
(iv) the following inequality holds:

$$M := (\theta_{sn}I_N - P - c\theta_l L) < 0 \quad (6)$$

where

$$\theta_{sn} = \theta_f + \frac{1}{2}\left(\sigma_m^2\theta_g^2 + \left(\sigma_c\theta_h\lambda_{max}(L^*)\right)^2\right),$$

then network (1)-(4) is global mean-square pinning controlled onto $X_r(t)$.

The complete proof of Theorem 1 is presented in [36]. A sketch of this proof is the follows. First, a candidate control Lyapunov function is considered for the controlled network (1)-(4), as

$$V(e) = \sum_{i=1}^{N} e_i^T e_i/2.$$

After, the differential operator $\mathscr{L}$ is applied to V(e), according to the Definition 3

$$\mathcal{L}V(e) = \sum_{i=1}^{N}\Bigg[e_i^{\mathrm{T}}\big(f(x_i) - f(x_r) - c\sum_{j=1}^{N} L_{ij}h(x_j) + u_i\big) + \frac{\sigma_m^2}{2}\Big(g(x_i) - g(x_r)\Big)^{\mathrm{T}}\Big(g(x_i) - g(x_r)\Big) + \frac{\sigma_c^2}{2}\Bigg(\Big(\sum_{j=1}^{N} L_{ij}^{*}h(x_j)\Big)^{\mathrm{T}}\sum_{j=1}^{N} L_{ij}^{*}h(x_j)\Big)\Bigg].$$

Following steps mathematical procedure appropriately, one obtains the next bounds:

(a)

$$\sum_{i=1}^{N} e_i^T(f(x_i) - f(x_r) + u_i) \le e^T[(\theta_f I_N - P) \otimes I_n]e.$$

(b)

$$-\sum_{i=1}^{N} e_i^T c \sum_{j=1}^{N} L_{ij}h(x_j) \le c\theta_l e^T(L \otimes I_n)e.$$

(c)

$$\sum_{i=1}^{N} \frac{\sigma_m^2}{2}\|g(x_i) - g(x_r)\|^2 \le \frac{\sigma_m^2\theta_g^2}{2}e^T e.$$

(d)

$$\sum_{i=1}^{N} \frac{\sigma_c^2}{2}\|\sum_{j=1}^{N} L_{ij}^{*}h(x_j)\|^2 \le \frac{1}{2}\big(\sigma_c\theta_h\lambda_{max}(L^*)\big)^2 e^T e.$$

By combining bounds (a)-(d), we obtain

$$\mathcal{L}V(e) \le e^T[(\theta_{sn}I_N - P - c\theta_l L) \otimes I_n]e = e^T(M \otimes I_n)e \le \lambda_{max}(M)\|e\|^2$$

Since $M < 0$, by selecting $\beta_1 = \beta_2 = 1$ and $\beta_3 = -\lambda_{max}(M)$, we have that (5) holds. Therefore, from Lemma 1, we can conclude that the trivial solution $e(t) = 0$ of the synchronization error dynamics (3) is mean square stable in probability.

Condition (i) is a common requirement for the vector field $f(\cdot,\cdot)$ in the framework of complex networks [22], several groups of dynamical systems including chaotic systems such as Lorenz system, Chen system, Lü system, and Chua's circuit satisfies this assumption. Conditions (ii)-(iii) are little conservatives and have been used in the literature on the control of stochastic network dynamical systems [30], [37]. Condition (iv) is related to the stability of the network where θ_{sn} clusters the influence of dynamics nodes (θ_f), noise dynamics ($\sigma_m\theta_g$), and noise presenting by flows ($\sigma_c\theta_h\lambda_{max}(L^*)$). Additionally, it is observed that the network topology ($c\theta_l L$) affects stability. If the uncontrolled network ($\psi = 0$) does not synchronize at the desired state, it is possible to design a pinning controller based on condition (iv), where the control problem is to determine the control Matrix P, i.e, determining how many and which nodes should be chosen for pinning, subject to the condition that the characteristic matrix M is negative definite. In what follows, we recall a necessary condition on the number Q of nodes to be pinned, a necessary condition on the coupling gain c and a sufficient condition on the parameter ψ such that $M < 0$.

Theorem 2: [21] If the matrix $M_{open} = \theta_{sn}I_N - c\theta_l L$ has $p > Q$ non-negative eigenvalues, then, for all $\psi \in \mathbb{R}, M \not\prec 0$.

The theoretical analysis of Theorem 2 is not difficult, the proof follows the same steps according to [21, Proposition 7.1], for this reason, it is omitted its demonstration. The utility of this result is to determine the minimal quantity of pinned nodes.

Now, Matrix M can be decomposed as

$$M = \begin{bmatrix} A - \psi I_Q & B \\ B^T & \widetilde{M} \end{bmatrix}$$

where A and B are matrices with appropriate dimensions, and then we leverage the Schur complement [38] to prove the following:

Lemma 2: If $\widetilde{M} < 0$ and $\psi > \lambda_{max}\big(A - B\widetilde{M}^{-1}B^T\big)$, then $M < 0$.

Proof: Since $\psi > \lambda_{max}\big(A - B\widetilde{M}^{-1}B^T\big)$, we have that $A - \psi I_Q - B\widetilde{M}^{-1}B^T < 0$. From the Schur complement [38], the hypothesis that $\widetilde{M} < 0$ yields the thesis.

Note that this result determines how to design the parameter ψ for the pinning controller (4).

Corollary 1: Let Conditions (i)-(iii) of *Theorem 1* hold. The stochastic network (1)-(4) is mean-square pinning controllable onto $x_r(t)$ if $\psi > \lambda_{max}\big(A - B\widetilde{M}^{-1}B^T\big)$, and $\theta_{sn} - c\theta_l\lambda_{min}\big(\tilde{L}\big) < 0$, where $\tilde{L}$ is the matrix obtained by removing the first Q rows and columns from the Laplacian L.

Proof: First, let us note that the generic element ij of matrix $M_{open} = \theta_{sn} I_N - c\theta_l L$ can be written as:

$$m_{ij} = \begin{cases} \theta_{sn} - c\theta_l L_{ii}, \text{ if } i = j, \\ -c\theta_l L_{ij}, \text{ otherwise.} \end{cases} \quad (7)$$

Remembering that the element L_{ii} in a graph is the degree of node i, defined as

$$d_i = L_{ii} = -\sum_{j=1, j\neq i}^{N} L_{ij}$$

A necessary condition for M_{open} being negative definite is that all of its diagonal entries are negative, therefore a necessary condition on the coupling gain c is

$$c > \frac{\theta_{sn}}{\theta_l d_i},$$

i.e,

$$c > \frac{\theta_{sn}}{\theta_l \lambda_{min}(\tilde{L})},$$

The other results, from *Theorem 1* and *Lemma 2*, the thesis follows.

Note that *Theorem 1* and *Corollary 1* can be used to assess mean-square pinning controllability, and a sufficient number of nodes that need to be pinned can be identified.

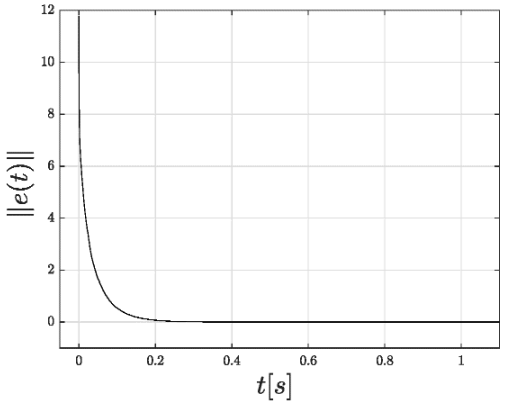


Fig. 2 *Time signal of the synchronization error norm ‖e(t)‖, for whole supply chain network.*
Source: *Authors.*

4. SIMULATION RESULTS

4.1 Stochastic Chaotic Supply Chain Network

The coupling and diffusion functions are selected as $h(x_i, t) = g(x_i, t) = x_i(t)$, respectively; the coupling strength is $c = 33$. The structure of the supply chain network is based on the case of study of Cardoso et al [39], which is a European supply network with 37 nodes, including 11 raw material suppliers, 3 factories, 5 warehouses and 18 markets. The topology describing the deterministic and noisy links among the nodes is illustrated in Fig. 1. We assume $\sigma_m = \sigma_c = 0.1$, modulating the intensity of noise in the node dynamics and communication links. The control goal of signal $u_i(t)$ is to synchronize network (1) onto $x_r(t)$, which is the trajectory of an isolated node.

The parameter θ_f for this case can be calculated using the same process in [22], $\theta_f = 28$. Additionally, for the coupling function $h(x_i, t)$ we obtain $\theta_l = 1$, $\theta_h = 1$ and $\theta_g = 1$. Note that $\lambda_{max}(L^*)^2 = 78.8$ and $\theta_{sn} = 28.4$. Calculating the cardinality p of the set $\{\lambda \in spec(\theta_{sn} I_N - c\theta_l L) : \Re(\lambda) \leq 0\}$, the minimal number the pinned nodes is eight. We select the nodes 12-19 as pinned with correspond with the 3 factories and the 5 warehouses. With this selection, we verify that $\tilde{M} < 0$ using Lemma 2, after calculate the bound for control parameter ψ using Corollary 1, which is $\psi > 747.7$. Selecting $\psi = 750$, the main condition of Theorem 1 is satisfied, i.e., the matrix M is negative definite.

The proposed supply chain network is simulated for 100 different random initial conditions, later, the results are averaged. As displayed in Fig. 2, using the pinning control with the design parameters, the closed-loop network fulfills the stability conditions established in the analytical results, i.e., the synchronization error norm converges exponentially, satisfying the definition of mean-square stability. The proposed scheme can help supply chain managers to select the appropriate control strategies to guarantee resiliency allowing to reduce the over costs and satisfy the demand over the different nodes.

5. CONCLUSIONS

This article looks at how to guarantee resilient recovery of a stochastic chaotic supply chain network in the face of problems disturbance or disruption. The network model proposed is affected by two different types of noise. Resilient recovery of supply network has been defined as synchronizing process between actual network and scheduled network. This objective is achieved through a pinning control law. A methodology for the design of the pinning controller is presented. The simulation results illustrate the effectiveness of the proposed approach.

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