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State observer control for robotic series elastic actuators

Control por observador de estados para actuadores elásticos series robóticos

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Abstract: This implementation shows the results through a control state observer strategy for a series elastic actuator (SEA) of motorized robotic limbs, with the purpose of evaluate the disturbance response with regard to convetional PID controller. By implementing a Matlab® script, the goal was to evaluate the state observer controller for a series elastic actuator space state dynamic model in comparison with a conventional PID controller, verifying the behavior of each by adding disturbances. The advantages of the state observer control strategy with respect to the conventional PID controller could be verified. The state observer ensures that the series elastic actuator is not affected by disturbances, presenting better performance than the PID in SEA control. The Matlab® simulation of the state space dynamic model of the series elastic actuator and its control by a state observer, shows the advantage that it offers in its dynamic response in scenarios with disturbances and eliminates its effect on the process in steady state.

Keywords: state observers, series elastic actuator, disturbance rejection control.

Resumen: Este artículo presenta los resultados obtenidos mediante la estrategia de control basada en observador de estado para un actuador elástico en serie (SEA) para cualquier extremidad robótica, con el propósito de evaluar la respuesta a la perturbación respecto a un controlador PID convencional. Mediante la implementación de un script en Matlab® el objetivo fue evaluar un controlador por observador de estado para el modelo de espacio de estados de un actuador elástico serie, en comparación con un controlador PID convencional, verificando el comportamiento de cada uno frente a la presencia de perturbaciones. Se pudieron comprobar las ventajas de la estrategia de control por observador de estado respecto al controlador PID convencional. El observador de estado asegura que el actuador elástico serie no se vea afectado por perturbaciones, presentando mejor desempeño que el PID convencional. La simulación en Matlab® del modelo dinámico en espacio de estados del actuador elástico en serie y su control por un observador de estado, mejora la respuesta dinámica en escenarios con perturbaciones y elimina su efecto sobre el proceso en estado estacionario.

Palabras clave: observadores de estado, actuador elástico serie, control de rechazo de perturbaciones.

1. INTRODUCTION

The intention of the research is aimed at exploring tools and alternative solutions, which provide new strategies that improve the performance of the actuating robotic mechanisms. This is based on models of dynamic systems of actuators applied, seeking to guarantee better performance and robustness against disturbances, in such a way that they give naturalness to the robotic movement, with better manageability performance.

One of the important components in the robotics is the actuator, which is responsible for providing the force and movement necessary to execute a natural movement. The series elastic actuator (SEA) is an emerging technology that has shown promise for application in robotics. SEA uses a series of elastic elements to mimic the function of human natural muscles, providing improved dynamic response and greater energy efficiency in robotic limbs [1].

Regarding the control systems for robotic actuating mechanisms, particularly in the case of SEA, many strategies have been used within the framework of classical control or conventional (PID) by the mechanical impedance control method, applied in all types of robot mobile components, however we want to explore other control strategies, with a view to seeking potential improvements in performance and response to disturbances in the case of elastic actuators applications like motorized robotic limbs [2].

1.1. Series Elastic Actuator

The function of actuators is to generate a force to move a mechanical type device. This generated force can come from three possible ways: pneumatic, hydraulic and electrical. The name of the source characterizes the type of actuator, and correspond to the force that is generating the action [3].

1.2. Output impedance

The mechanical output impedance is "the minimum force F that an actuator generates to achieve X movement in the load" [4]. Which in theory it should be zero. It is related to the output stiffness of the actuator. It has like transfer function:

$$Z(s) = \frac{F(s)}{X(s)} \quad (1)$$

1.3. Impedance Control

The goal of a control system is to obtain the desired setpoint of a process. The controller modifies the parameters in the system (actuator), so that the signal states remain in their normal state in the event of any variation or disturbance [5].

For elastic actuators, mechanical impedance control is recommended, used to control the SEA mechanism in the study and development of motorized robotic limbs. Proportional type controllers have been conventionally implemented and Integral and Derivative (PID) [6].

The position error signal is received by an impedance controller, which calculates a reference torque necessary to correct this error. Next is the torque control and finally the speed control. This method has been used to control different types of actuators with flexible and active elastic actuator devices. Achieve proper movement during robotic limb movement. [7], [8].

"The SEA is ideally a force source with zero mechanical output impedance, for this reason the actuator dynamics would be completely decoupled from the load". But the actuator is a real system with some value of output impedance. "The output impedance model is equivalent to a physical mass with mechanical friction, in series with a spring, driven by the movement of the load." [9], [10].

2. METHODOLOGY

The SEA mechanical construction includes an elastic spring element between the gearmotor and the load, which allows decoupling the load from the inertias and non-linear frictions of the motor and transmission and isolates the motor and transmission from external impacts and shocks [11], [12].

SEA have advantages over rigid conventional actuators in the design of motorized robotic limbs applications, as they have high controllability and force/torque capability, low mechanical output impedance and tolerance to shocks and disturbances [13], [14].

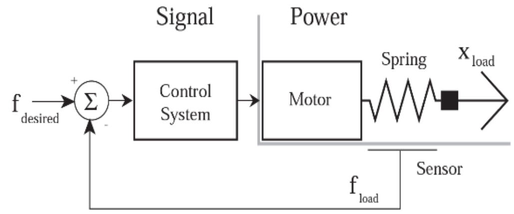

***Fig. 1**. SEA actuator components.*
***Source:** own elaboration.*

The figure 1, shows the SEA components separated by power and signal domain [10], [15]. The main elements are:

- PID Controller
- DC brushless Motor
- Elastic element (Spring)
- Sensor

Where three main blocks are identified like, brushless motor, transmission and spring, which correspond to a second-order mass mechanical system model, mass, spring and shock absorber [16].

2.1. SEA Actuator Brushless Motor

The brushless motor is the element that provides energy to the SEA, its model can be developed in two parts, an electrical model and a mechanical model, as illustrated in figure 2. In the electrical part it has an armature resistance R_a, armature inductance L_a and counter electromotive force v_b. In the mechanical part, a constant motor friction f and motor inertia J [9].

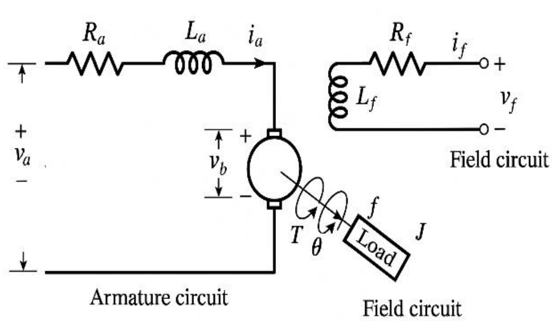

***Fig. 2**. Brushless Motor.*
***Source:** own elaboration.*

2.2. Brushless Motor Electrical Modeling

In a brushless motor, the torque τ is related to the motor armature current i_a of the winding given by:

$$\tau = K_\tau i_a(t) \quad (2)$$

The back electromotive voltage of the brushless motor $v_b(t)$ is related to the angular speed by:

$$v_b(t) = K_b \frac{d\theta}{dt} \quad (3)$$

Applying Kirchhoff's Voltage Law to the electrical circuit of the brushless motor:

$$R_a i_a(t) + L_a \frac{i_a(t)}{dt} + v_b(t) = v_a(t) \quad (4)$$

Replacing $v_b(t)$:

$$R_a i_a(t) + L_a \frac{i_a(t)}{dt} + K_b \frac{d\theta}{dt} = v_a(t) \quad (5)$$

Applying the Laplace transform to the equation that models the brushless motor electrical system we obtain:

$$R_a i_a(s) + L_a S i_a(s) + K_b S\theta(s) = v_a(s) \quad (6)$$

2.3. Brushless Motor Mechanical system model

It corresponds to the rotational mechanical system of the motor constituted by an inertia J_m and a friction K_f:

$$\tau(t) = J_m \frac{d^2\theta(t)}{dt^2} + K_f \frac{d\theta}{dt} \quad (7)$$

Replacing the electrical equivalence for motor torque $\tau = K_\tau i_a(t)$:

$$K_\tau i_a(t) = J_m \frac{d^2\theta(t)}{dt^2} + K_f \frac{d\theta}{dt} \quad (8)$$

Applying Laplace transform:

$$K_\tau i_a(s) = J_m S^2\theta(s) + K_f S\theta(s) \quad (10)$$

Solving for $i_a(s)$:

$$i_a(s) = \frac{J_m S^2\theta(s) + K_f S\theta(s)}{K_\tau} \quad (11)$$

2.4. Overall Brushless Motor model

Taking the last equation and replacing:

$$(R_a + L_a S) i_a(s) + K_b S\theta(s) = v_a(s) \quad (12)$$

$$(R_a + L_aS)\left[\frac{J_mS^2\theta(s)+K_fS\theta(s)}{K_\tau}\right] + K_bS\theta(s) = v_a(s) \quad (13)$$

Taking Common factor and adding terms:

$$\left[\frac{S(R_a + L_aS)(J_mS + K_f) + K_\tau K_bS}{K_\tau}\right]\theta(s) = v_a(s) \quad (14)$$

The system transfer function for position:

$$\frac{\theta(s)}{v_a(s)} = \frac{K_\tau}{S[(R_a + L_aS)(J_mS + K_f) + K_\tau K_b]} \quad (15)$$

To obtain the transfer function of the brushless motor for angular speed, the integral term 1/s is canceled:

$$\frac{\omega(s)}{v_a(s)} = \frac{K_\tau}{(R_a + L_aS)(J_mS + K_f) + K_\tau K_b} \quad (16)$$

Where J_m is the moment of inertia of the brushless motor [kg·m^2], K_f is the damping constant of the mechanical system or viscous friction coefficient of the \textit{brushless} motor [N·m·s], K_τ is the torque constant of the \textit{brushless} motor [N·m/A], K_b is the electromotive force constant [V/rad/s}, R_a is the resistance of the armature coil [Ω] and L_a is the inductance of the armature coil [H].

2.5. SEA Overall Model

From the system transfer function for position. Since the inductance has much smaller values than the other constants, $L_a=0$ is assumed, the model is simplified:

$$\frac{\theta(s)}{v_a(s)} = \frac{K_\tau}{S[(J_mR_aS + K_fR_a) + K_\tau K_b]} = \frac{K_m}{S(\tau_mS + 1)} \quad (17)$$

Where the motor constants are:

$$K_m = \frac{K_\tau}{K_\tau K_b + K_fR_a} \quad (18)$$

$$\tau_m = \frac{J_mR_a}{K_\tau K_b + K_fR_a} \quad (19)$$

The effect of transmission and load equivalent model is now included:

$$K_m = \frac{K_\tau}{K_\tau K_b + f_eR_a} \quad (20)$$

$$\tau_m = \frac{J_eR_a}{K_\tau K_b + f_eR_a} \quad (21)$$

Where:

$$J_e = J_m + J_L\left(\frac{n\cdot p}{2\pi}\right)^2 \quad (22)$$

$$f_e = K_f + f_L\left(\frac{n\cdot p}{2\pi}\right)^2 \quad (23)$$

Where $n=N_1/N_2$ is the transmission rate. The term $n\cdot p/2\pi$ corresponds to the conversion of the linear displacement of the motorized robotic limb into the rotation about the motor shaft generated by the transmission, and $(n\cdot p/2\pi)^2$ are the inertia and viscous friction components of the device acting on the load, reflected in the motor shaft.

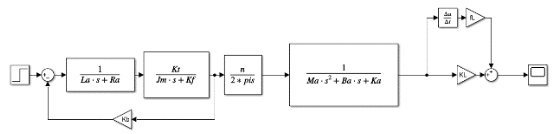

***Fig. 3**. Block Diagram of the SEA Overall Model.*
***Source:** own elaboration.*

2.6. Model Parameters

From the literature on research developed on the topic, a database is obtained for the constant values of the model [7]. The spring stiffness constant is the parameter that best characterizes the actuator and defines its dynamic behavior. The values are taken from the literature and illustrated in table 1, information like: motor mechanisms and transmission, manufacturers technical sheets, etc.

***Table 1:** SEA Model Parameters*

Parameter	Value	Units
Moment of inertia of the rotor (Jm)	0.01	kg · m2
Motor torque constant (Kτ)	0.01	N · m/A
Motor viscous friction constant (Kf)	0.1	N · s/m
Electromotive force constant (Kb)	0.01	v · s/rad
Electric resistance (Ra)	1	Ω
Electric inductance (La)	0.2	H
Load inertia constant (JL)	0.1	kg · m2
Load friction constant (fL)	0.1	N · s/m
Load stiffness constant (KL)	0.8	N/m
SEA mass (Ma)	2	Kg
SEA friction constant (Ba)	0.1	N · s/m
SEA stiffness constant(Ka)	0.8	N/m
Ballscrew pitch (p)	0.005	M
Transmission ratio (n)	0.001	-

***Source:** own elaboration.*

Figure 4 shows the block diagram representation of the actuator including the motor model, the model of the mechanical part of the SEA and the model of the load effects. By simplifying and replacing the constant values, the final model in figure 4 is obtained. For the brushless motor model:

$$G(s) = \frac{0.099}{0.099S^2 + S} \qquad (24)$$

For SEA actuator mechanical model:

$$G(s) = \frac{1}{2S^2 + 0.1S + 0.8} \qquad (25)$$

For the load effect model:

$$G(s) = 0.1s + 0.8 \qquad (26)$$

Finally, the final transfer function is obtained with parameters that represent the SEA model, considering each of its components to later apply the impedance control:

$$\begin{aligned} G(s) &= \frac{F(s)}{X(s)} \\ &= \frac{0.0099S + 0.079}{0.19S^4 + 2.01S^3 + 0.17S^2 + 0.8S} \end{aligned} \qquad (27)$$

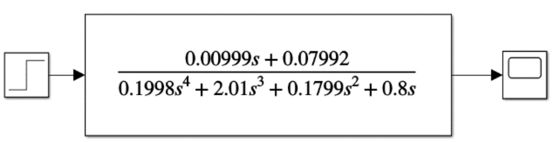

Fig. 4*. Final Model of SEA with Parameters.*
Source: *own elaboration.*

2.7. SEA Space State Representation

The formal representation of the equation of state is shown in figure 5.

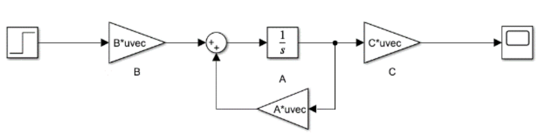

Fig. 5*. Speed response of the PID controller and the fuzzy logic controller.*
Source: *own elaboration.*

Which corresponds to the matrix representation:

$$\dot{X} = AX + BU \qquad (28)$$

$$Y = CX \qquad (29)$$

Taking as a starting point the transfer function of the systems equation

$$\begin{aligned} 0.19S^4y(s) + 2.01S^3y(s) + 0.17S^2y(s) \\ + 0.8Sy(s) \\ = 0.0099Su(s) \\ + 0.079u(s) \end{aligned} \qquad (30)$$

The differential equations are obtained in terms of Laplace:

$$\begin{aligned} S^4y(s) + 10.1S^3y(s) + 0.85S^2y(s) + 4.02Sy(s) \\ = 0.049Su(s) \\ + 0.039u(s) \end{aligned} \qquad (31)$$

Taking the inverse transform of Laplace with zero initial conditions:

$$\begin{aligned} y^{(4)} + 10.1\dddot{y} + 0.85\ddot{y} + 4.02\dot{y} \\ = 0.049\dot{u} + 0.039u \end{aligned} \qquad (32)$$

The variable change to system states is made:

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= x_3 \\ \dot{x_3} &= x_4 + 0.049u \\ \dot{x_4} &= -4.02x_2 - 0.85x_3 - 10.1x_4 \\ &\quad - 0.1u \end{aligned} \qquad (33)$$

Finally, the representation of the state space equation of the SEA is obtained:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -4.02 & -0.85 & -10.1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.049 \\ -0.1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad (35)$$

Figure 6 shows the space state representation based on the equations of system. The formal space state representation according to equation 35, where the matrices A,B and C are obtained.

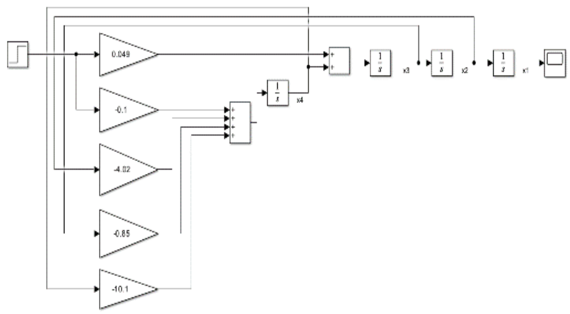

Fig. 6. *SEA Model Representation of State by Equations.*
Source: *own elaboration.*

3. DISCUSSION

3.1. SEA PID controller

The PID control of the SEA is simulated at a step input and the position of the actuator is plotted as a function of time, as well as the values of the state variable in the PID controller. The system parameters and transfer function are defined in the same way as in the code. Then the PID controller is defined using the PID function and with the Matlab® Sisotool.

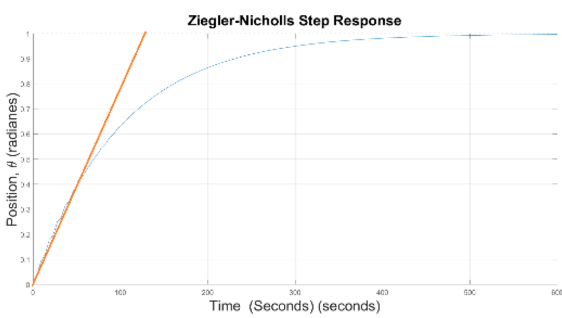

Fig. 7. *SEA PID reaction curve.*
Source: *own elaboration.*

In this case, for the SEA, given the dynamics presented by the system, the first Ziegler-Nicholls method or the reaction curve according to figure 7 was used. From the curve it is approximately obtained that *L=5* with *T=120* and *K=1* and control response according to figure 8.

$$a = \frac{KL}{T} = \frac{5}{120} \quad (36)$$

Integral time:

$$T_i = 2L = 10 \quad (37)$$

Derivative time:

$$T_D = \frac{L}{2} = \frac{5}{2} \quad (38)$$

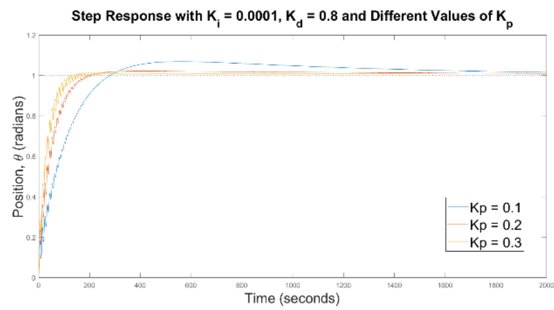

Fig. 8. *SEA PID tuning.*
Source: *own elaboration.*

3.2. Pole Placement and State Feedback

The presented pole location algorithm by feedback of state variables is implemented in Matlab®, the controllability of the model. The location of the poles is defined arbitrarily in a region on the left side of the S-plane, which guarantees a design with low overshoot and high speed stability, with four conjugate poles of value s+2±0.5i the gain being then calculated. The steady state or DC gain error in closed loop is then corrected with the algorithm $\overline{N}$ verifying the final response of the controller, obtaining the response of the controller, with low overshoot and optimal stabilization response of the figure 9.

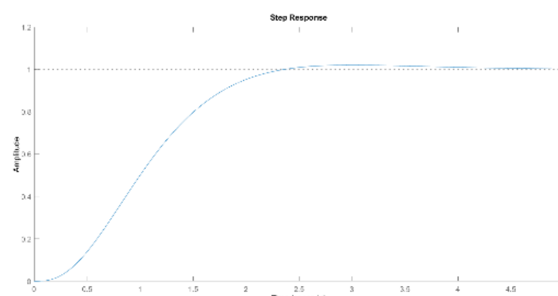

Fig. 9. *RVE controller response by pole placement.*
Source: *own elaboration.*

3.2. State Observer Results

The state observer is implemented in Matlab® in conjunction with the control gain calculated by the state variable feedback algorithm, the observability of the model is verified and the controller's response is checked. It is verified that the estimate presents the same response as the feedback control of state variables, in figure 11, which indicates that the

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estimator was correctly designed. Now the performance of the state observer is verified and compared with respect to the conventional PID controller. A constant disturbance is added to the PID control designed previously according to figure 10, observing the response in figure 11, it can be noted that the disturbance changes the state of the established reference value, generating a permanent deviation from the desired value, which is unacceptable behavior in the SEA.

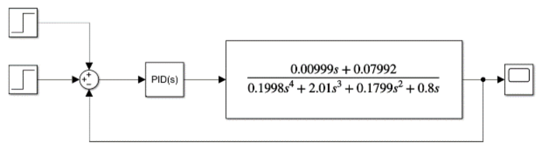

Fig. 10. *PID with added disturbance.*
Source: *own elaboration.*

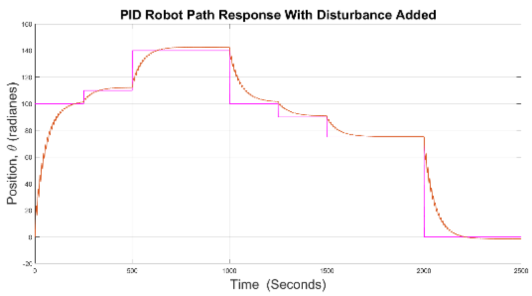

Fig. 11. *PID control response with disturbance added.*
Source: *own elaboration.*

Now the state observer is implemented according to the appendice code list, adding the disturbance, and it is verified in Simulink® according to figure 12 to obtain the response of the figure 13 where it can be seen that despite the disturbance entering, the observer quickly adjusts the response and restores the output of the SEA to the originally adjusted setpoint value.

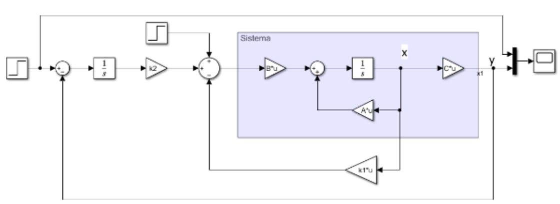

Fig. 12. *State Observer.*
Source: *own elaboration.*

It is clear according to each of the results achieved by testing the control strategies, that the state observer allows obtaining a response from the actuated SEA, which is within the design requirements regarding maximum overshoot and stabilization time, as well as, offering an alternative when disturbances occur in the system, reacting quickly to them and maintaining the original reference in the convenient ranges, which is reflected in better performance of the actuator with all the advantages in saving energy in motorized robotic limbs. Faced with the step-type disturbance, the actuator responds and stabilizes around the setpoint, attenuates it and keeps the steady state error close to zero.

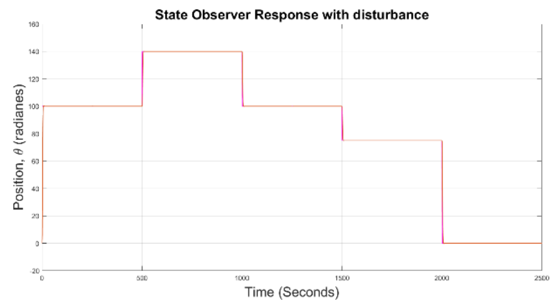

Fig. 13. *SEA State observer reponse*
Source: *own elaboration.*

It is observed that a reference tracking behavior of the setpoint established within the state variable feedback control scheme [17], allows the variable to remain in a previously configured operating state. The control signal that acts on the system does not exceed the expected limits of the actuator force according to the established setpoint, and meets all the objectives proposed for the system, in Matlab® code implementation in appendice code listing.

4. CONCLUSIONS

The Matlab® simulation of the state space dynamic SEA model of a motorized robotic limb and its control by a state observer and feedback of state variables as control gain, shows the advantage that it offers in its dynamic response in scenarios with disturbances and eliminates its effect on the process in steady state. Showing that this control strategy is a viable option compared to classic PID control, to improve the response of the SEA.

The impedance control transfer function of the SEA makes it easy to define the dynamics of the actuator, in such a way that it allowed different types of controllers to be verified from an initial PID controller, whose results have been significantly improved with the implementation of a controller by state observer and feedback of state variables.

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