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Detection of flight trajectory anomalies using autoencoders and Voronoi-based airspace segmentation

Detección de anomalías en trayectorias de vuelo utilizando autoencoders y segmentación del espacio aéreo basada en regiones de Voronoi

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Abstract: Given the increasing global air traffic, this article compares two autoencoder approaches for anomaly detection in flight trajectories, using the DBSCAN algorithm as an initial reference. The first model utilizes normalized continuous features (latitude, longitude, speed, and heading), while the second incorporates a discrete segmentation of the airspace through Voronoi regions, alongside kinematic variables. The results indicate on average 96% accuracy for the continuous autoencoder and 97% for the Voronoi-based model, with the latter showing a greater ability to identify normal trajectories. Qualitative analysis revealed that autoencoders, by including additional variables, capture more complex anomalies than DBSCAN. The integration of Voronoi regions improved the model's explainability, facilitating the interpretation of anomalies within their geographic context.

Keywords: anomaly detection, autoencoder, machine learning, unsupervised learning, voronoi regions.

Resumen: Dado el creciente tráfico aéreo mundial, este articulo compara dos enfoques de autoencoders para la detección de anomalías en trayectorias aéreas, empleando el algoritmo DBSCAN como referencia inicial. El primer modelo utiliza características continúas normalizadas (latitud, longitud, velocidad y rumbo), mientras que el segundo incorpora una segmentación discreta del espacio aéreo mediante regiones de Voronoi, además de las variables cinemáticas. Los resultados indican una precisión para la detección de anomalías en promedio del 96% en el autoencoder continuo y del 97% en el modelo basado en Voronoi, con este último mostrando una mayor capacidad para identificar trayectorias normales. El análisis cualitativo demostró que los autoencoders, al incluir variables adicionales, capturan anomalías más complejas que DBSCAN. La integración de Voronoi

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mejoró la explicabilidad del modelo, facilitando la interpretación de las anomalías en su contexto geográfico.

Palabras clave: detección de anomalías, autoencoder, machine learning, aprendizaje no supervisado, regiones de voronoi.

1. INTRODUCTION

The constant increase in global air operations, as observed in the data collected by Flightradar24 (see Fig. 1), places an increasing burden on air traffic control systems, which must ensure operational safety and efficiently manage airspace. As the volume of flights continues to rise, the implementation of automated systems to assist in decision-making becomes crucial to maintaining safety and efficiency levels.

In this context, the detection of deviations in typical aircraft trajectory characteristics, known as anomalies, can compromise the safety of air operations. Anomalies can arise due to various causes, such as adverse weather conditions [1], [2], technical failures, or human errors. Additionally, detecting these deviations can help reduce environmental impact by identifying irregularities that may increase fuel consumption and emissions of harmful gases [3].

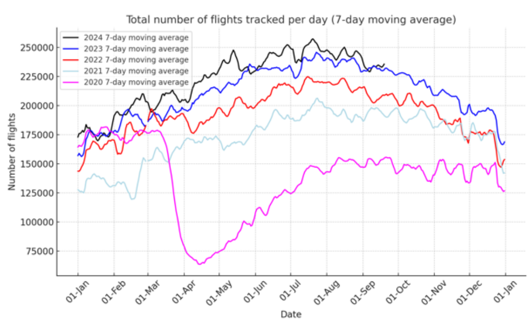

Fig. 1. *Global flights 2020–2024, data from Flightradar24.*
Source: *own elaboration.*

Additionally, early identification of anomalies can optimize the logistical costs associated with air operations. Anticipating deviations or delays allows for adjustments in ground management, such as gate assignment, operational staff, and resource allocation, thereby improving operational efficiency and reducing costs [3].

An anomaly is defined as any significant deviation from an expected pattern. From a statistical perspective, anomalies can be interpreted as shifts in the probability distribution within the data [4]. However, this deviation from "normal" also depends on the context, such as external factors like weather phenomena, traffic congestion during peak seasons, etc. Moreover, the low frequency of anomalous events in historical data poses another challenge. Nevertheless, the adoption of machine learning algorithms has enhanced the ability to detect anomalies in real-time across various industries and sectors [5], [6].

In aviation, an anomaly may refer to deviations in flight path, speed, altitude, or aircraft heading, among other available data. Various studies have addressed this topic using techniques such as autoencoders [7], [8], [9], [10], [11], generative adversarial networks (GANs) [8], support vector machines [3], and unsupervised algorithms like K-means [12], DBSCAN [13], and Isolation Forest [7], applied to different flight phases: takeoff, cruise, and landing.

The use of machine learning models facilitates a proactive safety approach by anticipating critical problems. This contrasts with traditional reactive approaches, where corrective measures are implemented only after an incident has occurred. Risk anticipation improves both safety and efficiency in air operations.

This article proposes a comparison and evaluation of two autoencoder-based approaches aimed at improving the accuracy of anomaly detection during the cruise phase of flight routes. The first approach works with continuous features, such as latitude, longitude, speed, and heading. The second approach uses a discrete representation of the airspace through Voronoi regions [14], along with continuous features of speed and heading. The main hypothesis of this article is that, by comparing both models quantitatively and qualitatively, the use of discrete Voronoi regions will provide a more compact and efficient representation of the airspace, enabling the detection of anomalies with comparable or even superior accuracy.

Furthermore, the Voronoi region-based approach enhances model explainability by allowing analysis of expected behavior within each specific region and how detected anomalies differ. This approach enables local interpretation of deviations, facilitating the identification and understanding of anomalous patterns, thereby improving operators' decision-making capability by providing a clear and precise visualization of where and how potential anomalies occur within the airspace.

The rest of the article is organized as follows: Section 2 describes the methodological phases used for the development of the research, including the algorithms employed and input data. Section 3 presents the results obtained, where anomaly detection approaches based on autoencoders are compared. Finally, Section 4 presents the conclusions and proposes future research directions related to improving the models.

2. METHODOLOGY

In this section, the methodology used for detecting anomalies in flight trajectories through a comparative approach of two autoencoders is described. As shown in Fig. 2, the methodological process begins with the preprocessing of flight data, followed by the identification of reference anomalies ("ground truth") using the DBSCAN algorithm, which classifies trajectory detections as normal or anomalous based solely on the spatial characteristics of latitude and longitude.

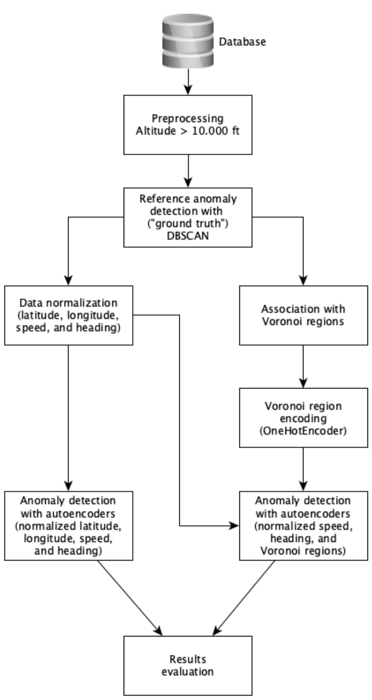

Fig. 2. *Methodology for anomaly detection.* ***Source:*** *own elaboration.*

Based on this classification, the autoencoders are trained exclusively with detections labeled as normal by DBSCAN. This allows the autoencoders to learn the typical behavior of flight trajectories to subsequently identify significant deviations that could represent anomalies.

The methodology is divided into two approaches: the first employs normalized continuous features (latitude, longitude, speed, and heading) to detect anomalies, while the second uses a discrete representation of the airspace through Voronoi regions, in addition to normalized speed and heading. Both approaches are evaluated and compared in terms of accuracy and anomaly detection capability, both quantitatively and qualitatively.

2.1. Database and Preprocessing

The database used consists of 120 flight trajectories of commercial flights on the Bogotá-San Andrés route, which connects El Dorado International Airport with Gustavo Rojas Pinilla International Airport. These data, retrieved from Flightradar24, cover the months of July and August 2024. Each flight is represented by a trajectory, which consists of a set of points recorded throughout the flight. These points, referred to as detections, capture information about the aircraft's position (latitude and longitude) and its kinematic behavior (altitude, speed, and heading).

The Bogotá-San Andrés route is one of the longest in Colombia, with an average duration of 90 minutes, which entails greater risks in the event of deviations or anomalies, as they can increase fuel consumption and emissions, in addition to compromising operational safety. The analysis focuses on the cruise phase of the flight; for this purpose, detections recorded below 10,000 feet (ft) in altitude were filtered out, as the takeoff and landing phases tend to introduce variations that do not accurately reflect the aircraft's behavior at stable altitude.

For the development of the proposed methodology, latitude and longitude were selected to represent the aircraft's position, while heading and speed were used as kinematic features. Altitude was excluded from the analysis due to its high correlation with speed, with a Pearson coefficient of 0.87, suggesting that both variables convey similar information. Therefore, it was determined that including altitude would be redundant.

2.2. Reference Anomalies (Ground Truth) with DBSCAN

To determine the ground truth of anomalies in flight trajectories, which will be used as a reference for the autoencoder algorithms in later stages, the DBSCAN algorithm (Density-Based Spatial Clustering of Applications with Noise) [15] was employed, using the implementation available in the scikit-learn library. This algorithm clusters nearby points based on their spatial density and classifies as anomalies those points that are scattered or isolated from the main clusters. DBSCAN was applied to the latitude and longitude coordinates to identify geographical deviations in flight trajectories.

In Fig. 3, a visualization of the results obtained with DBSCAN is presented. The green points correspond to detections classified as normal, while the red points represent the detected anomalies. Additionally, these anomalies will serve as a reference for evaluating the proposed models in the following stages.

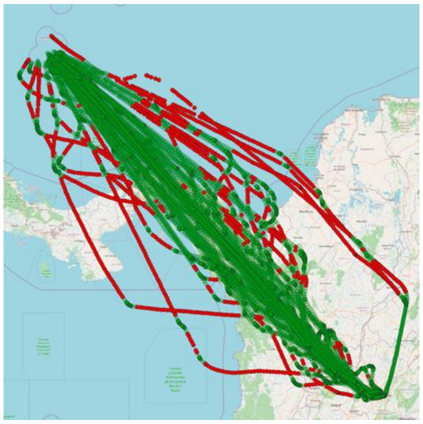

Fig. 3. *Normal detections and anomalies in flight trajectories identified by DBSCAN.*
Source: *own elaboration.*

The algorithm was configured with the following parameters:

- ϵ (epsilon): A value of 0.083 degrees was set, which corresponds to approximately 5 nautical miles, as the maximum radius within which two points are considered neighbors. This parameter controls the distance at which detections can belong to the same cluster.
- min_samples: Set to 10, meaning that at least 10 points must be within the ϵ radius for a point to be part of a group or cluster. For a detection not to be classified as an anomaly, it must belong to a cluster, regardless of which one. If a point does not meet this criterion, it is classified as noise or an anomaly.

2.3. Data Normalization

Prior to training the autoencoders, a normalization process was conducted using standard Z normalization, implemented in scikit-learn. This step is crucial because variables such as latitude, longitude, heading, and speed are on different scales, which could introduce biases into the model, favoring features with larger ranges. By normalizing the data, all features are adjusted to a uniform scale, allowing the model to treat all variables in a balanced manner.

Normalization adjusts each feature to have a mean of 0 and a standard deviation of 1, which is achieved using equation (1).

$$x_{normalizada} = \frac{x-\mu}{\sigma} \quad (1)$$

Where x is the original value of the feature, μ is the mean of that feature, and σ is the standard deviation.

2.4. Association of Detections with Voronoi Regions

To segment the airspace in the analysis of flight trajectories, Voronoi diagrams were used, following the methodology described in [14], which allows for spatial discretization by dividing the space into regions based on significant geodesic points, also known as generators.

For the Bogotá-San Andrés route, Voronoi regions were generated using the algorithm available in the QGIS geographic information system, referencing the significant points of the upper ATS routes in Colombia [15]. This process segments the airspace into a set of non-overlapping polygons.

As shown in Fig. 4, in areas of higher congestion, such as the center of the country around El Dorado Airport, there is a higher density of regions, reflecting the complexity of traffic management in these areas.

Each detection recorded during the flight is associated with a specific Voronoi region through a containment operation. This process adds a new categorical attribute to the trajectory data, referred to as the Voronoi zone.

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The Voronoi zone attribute allows detections from multiple trajectories to be grouped, reducing the continuous spatial complexity and facilitating the analysis of flight patterns. In this way, variability in trajectories is confined within a finite set of regions, simplifying the identification of anomalous patterns or significant deviations.

Fig. 4 shows an example of how Voronoi regions are generated using the significant points of the upper routes in Colombia.

Fig. 4. *Voronoi regions generated with the upper routes in Colombia.*
Source: *own elaboration.*

2.5. Encoding of Voronoi Regions

Once each detection was associated with its corresponding Voronoi region, the categorical regions were encoded using the One-Hot Encoding technique, implemented with scikit-learn. Since autoencoders do not directly support categorical variables, it is necessary to transform the Voronoi regions into a binary format that the model can process. This encoding converts each region into a column where the values are 0 or 1, indicating whether a detection belongs to a specific region or not.

Before performing the encoding, the most representative Voronoi regions for the Bogotá - San Andrés route was identified. For this purpose, a minimum threshold of 10 trajectories per region was set, so only those that exceeded this threshold were considered significant. Fig. 5 shows the distribution of trajectories per region. Regions that did not meet this threshold were excluded from the encoding, ensuring that detections within those zones do not influence the training of the autoencoder.

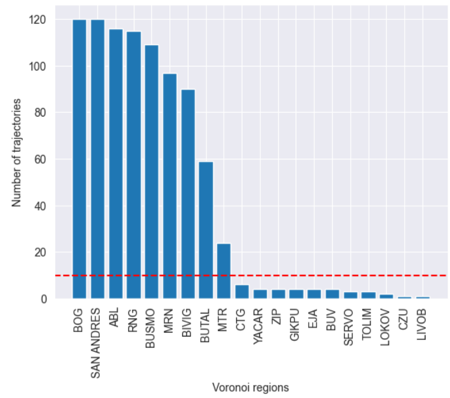

Fig. 5. *Trajectories per Voronoi region for the Bogotá - San Andrés route.*
Source: *own elaboration.*

During the prediction process, any Voronoi region not known in the training set is encoded as a zero vector. This approach ensures that less relevant regions or those outside the training set do not interfere with the model's performance, maintaining an appropriate categorical representation of the segmented airspace.

2.6. Anomaly Detection with Autoencoders

For the detection of anomalies in flight trajectories, two autoencoders were trained with different input configurations. The first one uses normalized continuous features (latitude, longitude, speed, and heading). The second incorporates a discrete representation of the airspace using Voronoi regions, encoded with One-Hot Encoding, along with the normalized kinematic features speed and heading. Both models were implemented using the Keras library, with an Adam optimizer and the mean squared error (MSE) loss function, defined in equation (2).

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{x}_i)^2 \qquad (2)$$

Where x_i is the original value, $\hat{x}_i$ is the reconstructed value, and n is the number of features. The goal of the autoencoders is to minimize the reconstruction error, allowing them to learn the normal patterns of the trajectories, and based on this, detect anomalies.

Both autoencoders were trained for 100 epochs with a batch size of 32. The hidden layers of both models use the ReLU activation function, and the output layer uses a linear activation function.

The architecture of both autoencoders is detailed in Table 1.

Table 1: *Autoencoder Architecture*

Layer	Continuous Autoencoder	Voronoi Autoencoder
Input	4	11
Encoder 1	16	64
Encoder 2	8	32
Bottleneck	4	8
Decoder 1	8	32
Decoder 2	16	64
Output	4	11

Source: *own elaboration.*

In Fig. 6, a comparison of reconstruction errors between both autoencoders is presented. The results show that the continuous autoencoder exhibits greater dispersion in reconstruction errors, reaching values up to 0.38. This suggests that the continuous model is more sensitive to variations in the trajectories or anomalies in the data, indicating greater difficulty in accurately reconstructing certain points.

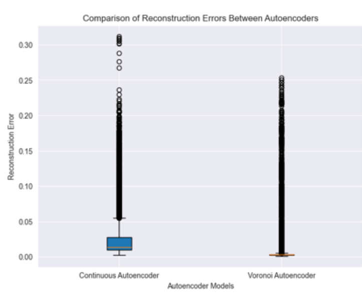

Fig. 6*. Comparison of reconstruction errors.*
Source: *own elaboration.*

On the other hand, the autoencoder based on Voronoi regions (Voronoi autoencoder) shows reduced dispersion in reconstruction errors, with a maximum of 0.25. This suggests that the Voronoi model provides greater consistency in its predictions. The difference in the behavior of both models could be attributed to the discrete segmentation of the airspace in the Voronoi approach, which simplifies the representation of the trajectories. Additionally, this model includes a larger number of input parameters by incorporating 9 Voronoi regions along with the speed and heading features. This increased complexity is reflected in the higher number of neurons in the encoder and decoder layers, allowing it to reconstruct trajectories more accurately and capture patterns.

3. RESULTS

To evaluate and compare the results obtained by both autoencoders (continuous and Voronoi), a quantitative and qualitative analysis was conducted. Different reconstruction thresholds were set for each model based on the analysis of reconstruction errors (Fig. 6). For the continuous autoencoder, a threshold of 0.02 was established, while for the Voronoi autoencoder, the threshold was 0.014. These thresholds determine when a detection is classified as anomalous or normal.

In the quantitative analysis, metrics such as precision, recall, and F1-score were examined, comparing the predictions of the autoencoders with the labels generated by DBSCAN. On the other hand, the qualitative analysis assessed how each approach segments and classifies the detections.

In quantitative terms, both autoencoders demonstrated high effectiveness in detecting anomalies compared to the reference labels generated by DBSCAN, as shown in Table 2. The autoencoder based on continuous features (latitude, longitude, speed, and heading) achieved on average 96% precision in detecting anomalies, while the autoencoder based on Voronoi regions achieved a slightly higher precision on average of 97%. However, the differences in identifying normal trajectories were significant: the continuous autoencoder obtained 8% precision, compared to 21% for the Voronoi-based model.

Table 2: *Quantitative Evaluation of Autoencoders*

Metric	Continuous Autoencoder	Voronoi Autoencoder
Precision (normal)	8%	21%
Precision (anomaly)	96%	97%
Recall (normal)	11%	32%
Recall (anomaly)	94%	95%
F1-score (normal)	9%	25%
F1-score (anomaly)	95%	96%
Macro avg	0.53	0.63

Source: *own elaboration.*

The recall, which measures the model's ability to correctly detect anomalies, was similar for both approaches, with values on average close to 95% for both the Voronoi and continuous autoencoders. However, the F1-score and macro average (the average between both classes: normal and anomaly) reflect an advantage for the Voronoi-based model, which achieved a macro average of 0.63 compared to 0.53 for the continuous autoencoder. This result indicates that the Voronoi autoencoder is not only more efficient at detecting anomalies but also more accurate in identifying normal trajectories, which is relevant for reducing false positives.

Since DBSCAN relied exclusively on latitude and longitude for anomaly detection, while the autoencoders consider additional features such as speed and heading, the quantitative metrics may be affected by the autoencoders' ability to detect

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relevant patterns that DBSCAN failed to identify. This underscores the need to complement the quantitative analysis with a qualitative analysis to assess cases where the models capture significant deviations not detected by DBSCAN.

The qualitative analysis revealed important differences in the behavior of the two autoencoders. In Fig. 7, the continuous autoencoder classifies as normal detections that are far from the expected air corridors, while the Voronoi-based approach shows greater control in classification, associating the detections with representative regions for the route (Fig. 8). In both cases, green points correspond to detections classified as normal, and red points represent anomalies. This suggests that the continuous autoencoder is more sensitive to small variations in trajectories, which can lead to incorrect classifications and a higher rate of false positives.

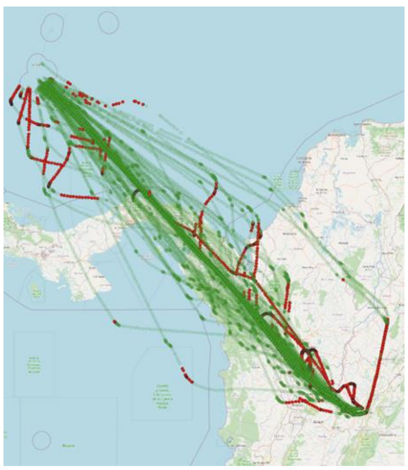

Fig. 7. *Anomalous and normal detections identified by the continuous autoencoder.*
Source: *own elaboration.*

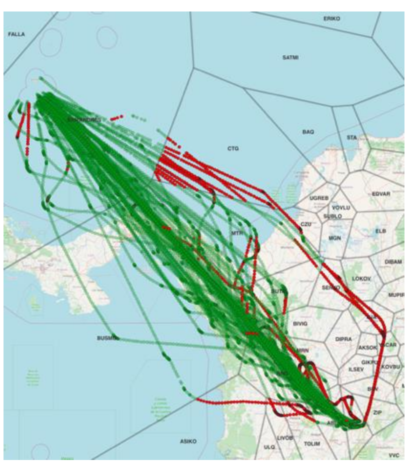

Fig. 8. *Anomalous and normal detections identified by the Voronoi autoencoder.*
Source: *own elaboration.*

A notable feature of the Voronoi Autoencoder is its ability to provide greater explainability in the results. By aligning with the segmentation of the airspace through Voronoi regions, it allows for clearer interpretation of where anomalies are in relation to the geographical space, thus facilitating the analysis and understanding of flight patterns. As seen in the visualization of trajectories by Voronoi region (Fig. 9), the model offers an intuitive representation of the distribution of anomalies throughout the airspace, providing a valuable tool for the qualitative analysis of the results.

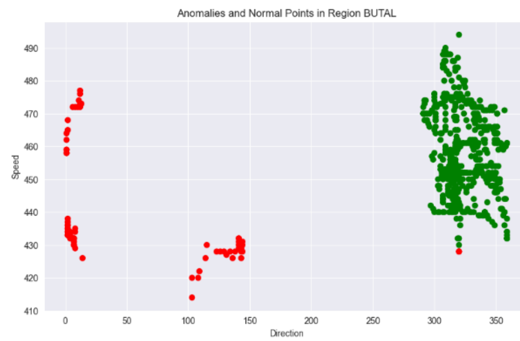

Fig. 9. *Anomalous and normal data identified by the Voronoi autoencoder for the BUTAL region.*
Source: *own elaboration.*

4. CONCLUSIONS

This article presents a comparison between two autoencoder approaches for anomaly detection in flight trajectories. The methodology uses the unsupervised DBSCAN algorithm to generate an initial reference of anomalies, which is then employed in the training of the autoencoders. These models differ in their input data: one is trained with normalized continuous features (latitude, longitude, speed, and heading), while the other incorporates a discrete representation of the airspace through Voronoi regions, along with the kinematic variables speed and heading.

Both autoencoders demonstrated high performance in anomaly detection, on average with 96% accuracy for the continuous autoencoder and 97% for the Voronoi autoencoder. However, significant differences were observed when evaluating the ability to identify normal trajectories based on the labels generated by DBSCAN. While the continuous autoencoder achieved 8% accuracy, the Voronoi model reached 21%.

By complementing the quantitative results with a qualitative analysis, as seen in Fig. 7 and Fig. 8, it can be inferred that, although DBSCAN is based on geographical distances, the autoencoders detect more complex anomalies by including kinematic variables. This allows detections to be classified not only by proximity but also by the aircraft's behavior in the airspace. This observation highlights the importance of incorporating expert evaluations to validate the results beyond quantitative metrics, a common challenge in the evaluation of unsupervised models, as noted by authors like [13].

The representativeness of the data is critical for training autoencoders, which rely on normal data to identify anomalies. In future work, a semi-supervised approach is suggested for constructing the ground truth, where expert intervention improves the selection of normal data and enhances the models' effectiveness.

A key aspect of this work is the redefinition of features through Voronoi regions, which segment the airspace in a way that is contextualized with operations. This segmentation allows for a more intuitive representation of the trajectories and a more detailed analysis of anomalies, offering greater clarity in the relationship between anomalies and spatial location, as demonstrated in Fig. 9.

Finally, the use of autoencoders in anomaly detection for flight trajectories supports a proactive safety approach. The ability to identify anomalous patterns before they become incidents contributes not only to improving operational safety but also to optimizing the efficiency of airspace management.

REFERENCES

[1] R. H. Cáceres León, “Meteorología aplicada a la seguridad de las operaciones aéreas,” *Ciencia y Poder Aéreo*, Jun. 2017, doi: https://doi.org/10.18667/9789585996113.

[2] J. Ortega, J. Florez, S. Lorduy, G. Jimenez, and O. Quintero, “Improve decision-making process in Air Command and Control Systems with meteorological data fusion,” in *2021 International Conference on Decision Aid Sciences and Application (DASA)*, IEEE, Dec. 2021, pp. 636–642. doi: 10.1109/DASA53625.2021.9682330.

[3] J. Mendling, C. Di Ciccio, H. van der Aa, C. Cabanillas, and J. Prescher, “Detecting flight trajectory anomalies and predicting diversions in freight transportation,” *Decis Support Syst*, 2016, doi: 10.1016/j.dss.2016.05.004.

[4] M. Ahmed, A. Naser Mahmood, and J. Hu, “A survey of network anomaly detection techniques,” *Journal of Network and Computer Applications*, vol. 60, pp. 19–31, Jan. 2016, doi: 10.1016/j.jnca.2015.11.016.

[5] R. A. Ariyaluran Habeeb, F. Nasaruddin, A. Gani, I. A. Targio Hashem, E. Ahmed, and M. Imran, “Real-time big data processing for anomaly detection: A Survey,” *Int J Inf Manage*, vol. 45, no. February, pp. 289–307, Apr. 2019, doi: 10.1016/j.ijinfomgt.2018.08.006.

[6] M. E. Villa-Pérez, M. Á. Álvarez-Carmona, O. Loyola-González, M. A. Medina-Pérez, J. C. Velazco-Rossell, and K.-K. R. Choo, “Semi-supervised anomaly detection algorithms: A comparative summary and future research directions,” *Knowl Based Syst*, vol. 218, p. 106878, Apr. 2021, doi: 10.1016/j.knosys.2021.106878.

[7] L. Coelho e Silva and M. C. R. Murça, “A data analytics framework for anomaly detection in flight operations,” *J Air Transp Manag*, vol. 110, Jul. 2023, doi: 10.1016/j.jairtraman.2023.102409.

[8] M. Aksoy, O. Ozdemir, G. Guner, B. Baspinar, and E. Koyuncu, “Flight trajectory pattern generalization and abnormal flight detection with generative adversarial network,” in *AIAA Scitech 2021 Forum*, American Institute of Aeronautics and Astronautics Inc, AIAA, 2021, pp. 1–11. doi: 10.2514/6.2021-0775.

[9] X. Olive and L. Basora, “Detection and identification of significant events in historical aircraft trajectory data,” *Transp Res Part C Emerg Technol*, vol. 119, Oct. 2020, doi: 10.1016/j.trc.2020.102737.

[10] M. Memarzadeh, B. Matthews, and T. Templin, “Multiclass Anomaly Detection in Flight Data Using Semi-Supervised Explainable Deep Learning Model,” *Journal of Aerospace Information Systems*, vol. 19, no. 2, pp. 83–97, Feb. 2022, doi: 10.2514/1.I010959.

[11] A. Chevrot, A. Vernotte, and B. Legeard, “CAE: Contextual auto-encoder for multivariate time-series anomaly detection in air transportation,” *Comput Secur*, vol. 116, May 2022, doi: 10.1016/j.cose.2022.102652.

[12] M. Y. Pusadan, J. L. Buliali, and R. V. Hari Ginardi, “Cluster Phenomenon to Determine Anomaly Detection of Flight Route,” *Procedia Comput Sci*, vol. 161, pp. 516–526, 2019, doi: 10.1016/j.procs.2019.11.151.

[13] S. J. Corrado, T. G. Puranik, O. P. Fischer, and D. N. Mavris, “A clustering-based quantitative analysis of the interdependent relationship between spatial and energy

anomalies in ADS-B trajectory data," *Transp Res Part C Emerg Technol*, vol. 131, Oct. 2021, doi: 10.1016/j.trc.2021.103331.

[14] Jose Ortega, Jimmy Florez, Mónica Hernández, and Jhon Escobar, "Trajectory Validation for Decision Making in Air. Traffic Management Using Voronoi Diagram," *Journal of Artificial Intelligence and Soft Computing Research*, vol. 12, no. 4, 2022, [Online]. Available: https://www.webology.org/data-cms/articles/20230215105345pmWEBOLOGY%2019%20(6)%20-%20164.pdf

[15] M. Ester, H.-P. Kriegel, J. Sander, and X. Xu, "A density-based algorithm for discovering clusters in large spatial databases with noise," in *Proceedings of the Second International Conference on Knowledge Discovery and Data Mining*, in KDD'96. AAAI Press, 1996, pp. 226–231.

[16] Unidad Administrativa Especial de la Aeronáutica civil (organization), "ENR 3.2 RUTAS ATS SUPERIORES ENR 3.2 UPPER ATS ROUTES," Aug. 2024, Accessed: Sep. 20, 2024. [Online]. Available: https://www.aerocivil.gov.co/servicios-a-la-navegacion/servicio-de-informacion-aeronautica-ais/Documents/AIP%20AMDT%2065_23/ENR/ENR%203.2.pdf