

Design and implementation of a state feedback control system for a Ball and Plate platform

Diseño e implementación de un sistema de control por retroalimentación de variables de estado para una plataforma Ball and Plate

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Abstract: This article will show the modeling, development and implementation of a LQR controller with integral action for an electromechanical plant with two freedom axes, in this case a “Ball and Plate”. A LQR controller with integral action greatly improves the system's response times, thus allowing its steady-state errors to be drastically reduced and providing a rapid response to applied external disturbances. This is optimal for application since it is necessary to have quick and precise control actions that will keep the ball in the desired reference position. This will be achieved through the acquisition of data from a two-axis resistive piezoelectric sensor, which delivers the data of the current position of the ball to an Arduino Mega whose function is to apply the implemented algorithms of reading, to later send the control actions to two MG-995 servo motors that will be responsible for altering the position of the ball by balancing it in the two axes of freedom. The Matlab software and its simulation tool Simulink will be used to obtain the different controller constants and simulate the control loop.

Keywords: Control, LQR controller, Ball and plate.

Resumen: El presente artículo mostrará el modelamiento, desarrollo e implementación de un controlador LQR con acción integral para una planta electromecánica de dos ejes de libertad, en este caso un “Ball and Plate”. Un controlador LQR con acción integral mejora en gran medida los tiempos de respuesta del sistema, permitiendo de esta forma reducir drásticamente sus errores en estado estacionario y provee una rápida respuesta ante perturbaciones externas aplicadas. Esto es óptimo para esta aplicación pues es necesario tener acciones de control rápidas y precisas que harán que la bola se mantenga en la posición de referencia deseada. Se utiliza para la adquisición de datos un sensor piezoeléctrico resistivo de dos ejes, el cual entrega los datos de la posición actual de la bola a un Arduino Mega que tiene como función aplicar los algoritmos implementados de lectura, para posteriormente enviar las acciones de control necesarias a dos servomotores MG-995 que se encargan de alterar la posición de la bola mediante el balanceo de esta en los dos ejes de libertad. Se utiliza el software Matlab y su herramienta de simulación Simulink para la

obtención de las distintas constantes del controlador y la simulación del lazo de control.

Palabras clave: Control, controlador LQR, Ball and plate.

1. INTRODUCTION

Ball and Plate is a two degree of freedom control system designed to maintain the position of a ball on a surface or plate at a desired reference point. To achieve this, it's necessary to control its movement in both X and Y degrees of freedom.

For this purpose, two independent systems can be devised for each axis, as proposed by [11], [2]. In this way, if the system is considered ideal and linear, a transfer function that describes the prototype's behavior will be founded. [3], [4]. This open loop transfer function will have the main characteristic of belonging to a second-order critically damped or also called oscillatory model. [5], [6]. From this transfer function, a representation of the system in state space is obtained. [7], [8], with this space state representation, the controllability of each system will be analyzed, which provides information about the viability of the implementation of the controller. [9]. Finally, begins the implementation of the LQR controller with integral action, which allows to generate a more precise and optimal control action unlike with conventional controllers. This control technique allows the large reduction of the oscillations present in the systems, in addition to increasing their efficiency [10], [11].

2. CONSTRUCTION AND DESIGN

For the design, where followed the guidelines proposed by [2], [12] for the modeling of the system and the obtention of a transfer function, first where analyzed the system dynamics, from the ball and going downward to the servomotors. These considerations are taken into account:

- The ball is rolling, not sliding.
- There's no friction on the system.
- ball geometry is totally spheric and homogeneous.

From this point, the analysis of the two axes X and Y was divided into two independent subsystems.

Figure 1 shows the considerations of each axis and the variables of the model.

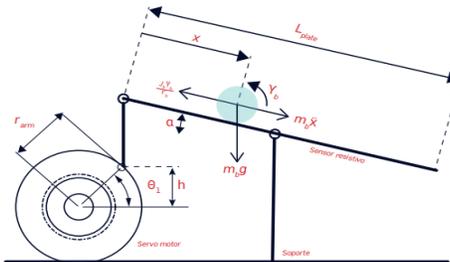


Fig. 1. Diagram for the analysis of the motion equations in each axis of the plant.

Source: One dimension modelling [13]

In this manner, two equations of motion for the ball are derived for each axis as follows:

$$\frac{d^2(x,y)}{dt^2} = \frac{m_b \cdot r_b \cdot (x \cdot \frac{d^2 \alpha_{1,2}}{dt^2} - g \cdot \text{sen} \alpha_{1,2})}{m_b \cdot r_b^2 + I_b} \tag{1}$$

As observed in equation (1), it is necessary to linearize the transfer function to eliminate the sinusoidal term from the dynamics. This is achieved by employing the Taylor series approximation for the sine function, which states that the sine of an angle can be approximated by the angle itself when the angle is close to 0. Angles of freedom ranging from -15 to 15 degrees are considered. [14]. Thus, it is possible to linearize the system for an operating range close to the center of the sensor, resulting in the following motion equation for the system (2):

$$\frac{d^2(x,y)}{dt^2} = \frac{m_b \cdot r_b^2 \cdot g \cdot \alpha_{1,2}}{m_b \cdot r_b^2 + I_b} \tag{2}$$

Based on this model of the plant, the following scheme must be followed presenting the total transfer function of the system including the internal transfer function of the servo motors.

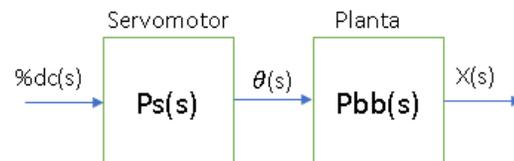


Fig. 2. Ball and plate open loop diagram.

Source: self-elaboration.

To obtain the transfer function, the measures outlined in the linearized system equation are first taken into account for obtaining the two transfer functions. These measures are presented in Table 1.

Table 1: Characteristics, parameters, and dynamics of the system.

Resistance	R	4	Ω
Inductance	L	2.75	μH
Electromotive Constant	Kb	0.0275	V/(rad/s)
Torque constant	Kp	0.0275	Nm/A
Inertia Constant	J	3.2284	p(Kg*m*m)
Friction constant	B	3.5077	u(Nms/rad)
Ball diameter	Rb	0.015	m
Ball mass	Mb	0.11	Kg
Servo arm longitude	d	0.017	m
Distance to base	Lp	0.073	m

With the information gathered from the previous table, the following transfer functions are finally obtained for both axes.

X axis:

$$X(s) = \frac{0.0103}{s^2}$$

Y axis:

$$Y(s) = \frac{0.006483}{s^2}$$

Figure 3 illustrates the final implementation of the physical system:

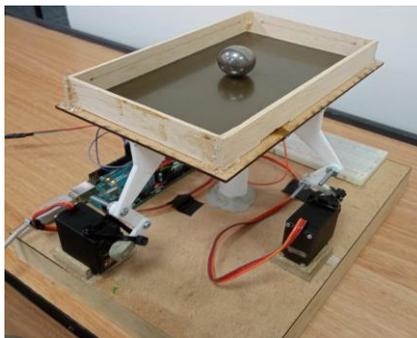


Fig. 3. Ball and Plate.
 Source: self-implementation.

3. STATE SPACE CONTROLLER

For the controller design, the first step is to obtain the equivalent state-space model. Starting with the two transfer functions obtained in section 2, the equivalent closed-loop transfer function is determined, and an equivalent state-space model is derived for each axis. [7].

For Y axis:

$$A_y = \begin{bmatrix} 0 & -0.0065 \\ 1 & 0 \end{bmatrix}$$

$$B_y = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C_y = \begin{bmatrix} 0 & 0.0065 \end{bmatrix}$$

For X axis:

$$A_x = \begin{bmatrix} 0 & -0.0103 \\ 1 & 0 \end{bmatrix}$$

$$B_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C_x = \begin{bmatrix} 0 & 0.0065 \end{bmatrix}$$

For both cases, a D matrix is determined to be equal to 0. To verify the operation of each axis in open-loop, both systems are implemented in Simulink, as shown in Figure 4.

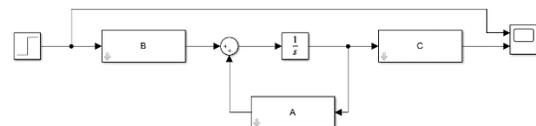


Fig. 4. Space state system representation
 Source: Self-Implementation.

Obtaining an oscillatory response from the system as observed in Figure 5, we can generalize this behavior, considering that both subsystems are critically damped.

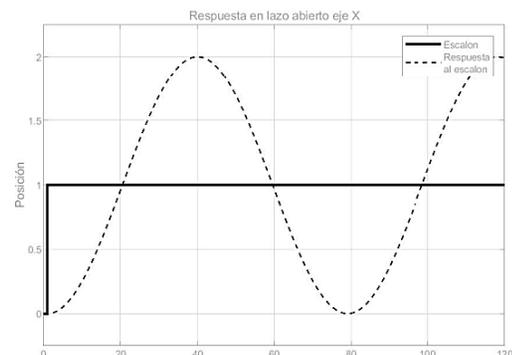


Fig. 5. Open loop system response.
 Source: Self-Implementation.

To continue with the controller design, the controllability of the system is now being studied. This will determine whether it's possible to implement a state feedback controller for this system. To find the controllability, we will determine the rank of the controllability matrix. If the rank is N, where N is the number of state variables of the system [5].

$$\theta_c = [B, (A * B)] \tag{3}$$

In (3), the method to determine the controllability matrix of the system is showed. From this, the controllability matrices for each axis are obtained as follows:

$$\theta_{cx} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \theta_{cy} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Given that both matrix have a rank of 2, both systems are controllable.

The design of a LQR controller with integral action is presented for each of the axes. To achieve this, the augmented matrix of the system are obtained in order to find the gain matrix and the integral gain of the controller [15].

The new state variable is a variable dependent on the error aiming for the steady-state error to be 0. To achieve this, we will have the following augmented matrix for each axis.

X axis:

$$A_{aux} = \begin{bmatrix} 0 & -0.0103 & 0 \\ 1 & 0 & 1 \\ 0 & 0.0103 & 0 \end{bmatrix}$$

$$B_{aux} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Y axis:

$$A_{aoy} = \begin{bmatrix} 0 & -0.0064 & 0 \\ 1 & 0 & 0 \\ 0 & 0.0064 & 0 \end{bmatrix}$$

$$B_{aoy} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Now, two additional poles must be assigned in the left part of the imaginary axis of the complex plane S. Since this is a critically oscillatory system, its poles are known to lie on the imaginary axis. Consequently, any negative value can be assigned. For this case, the pole matrix was assigned as follows:

$$Pcl = [-3, -4, -5]$$

With the proposed poles, the value of the gain matrix was determined, resulting in the following gains for each axis:

$$Gclx = [12, 46.989, 5825]$$

$$Gcly = [12, 46.999, 9230]$$

The first two values of the presented gain matrix will belong to the state variable gains, and the last value is the integral gain value of the controller.

Now, LQR controllers with integral actions are implemented in Simulink as shown in Figure 6:

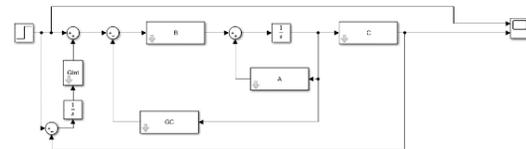


Fig. 6. Block diagram of the systems for LQR controller with integral action. Source: Self-Implementation.

And the following step responses are obtained for each axis as shown below:

X axis:

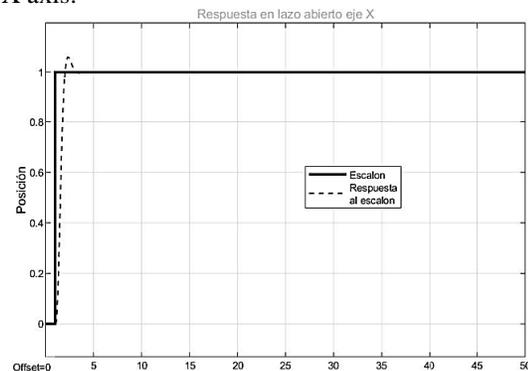


Fig. 7 Step response of the system with the implemented controller on the X-axis. Source: Self-Implementation.

Y axis:

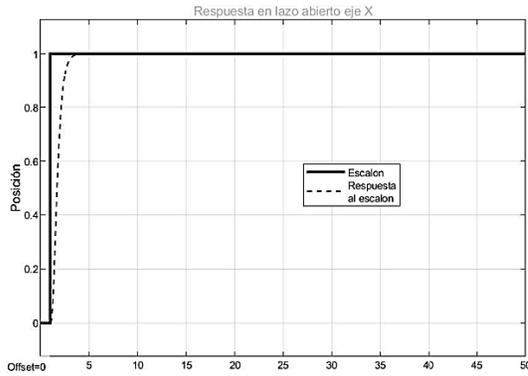


Fig. 8. Step response of the system with the implemented controller on the Y-axis.
 Source: Self-Implementation.

This completes the design of LQR controller with integral action.

4. RESULTS

To verify the controller's performance, various disturbances were applied to the system. The same simulation presented in Figure 6 was used, but disturbances were added to the output as shown in Figure 9.

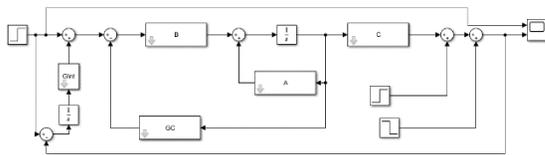


Fig. 9. Block diagram of LQR controller with integral action add disturbances.
 Source: Self-Implementation

Thus, a response to an impulse with disturbances was obtained as follows:

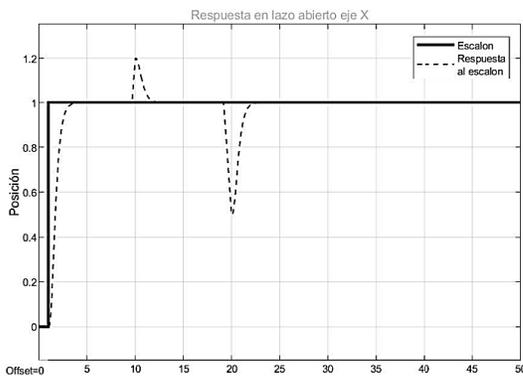


Fig. 10. Step response with disturbances of the system with the implemented controller on the Y-axis.
 Source: Self-Implementation

And for the other axis, we will have a response as follows.

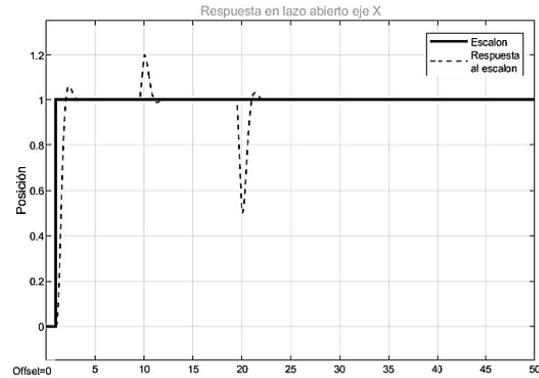


Fig. 11 Step response with disturbances of the system with the implemented controller on the Y-axis
 Source: Self-Implementation.

As observed in Figures 10 and 11, the system responds well to applied disturbances, whether positive or negative. This means that the system can effectively be perturbed in any direction on both axes, making it more robust and capable of correcting potential errors made during system modeling. Moreover, it suppresses almost all oscillations, unlike with other types of controllers.

Here are the constants of a conventional PD controller that was initially implemented in the physical model:

$$K_p = 20, K_d = 1.1$$

In this way, the two impulse responses can be compared to verify the speed, efficiency, and accuracy of the two types of controllers. Figure 12 shows the block diagram used in Simulink to compare both controllers.

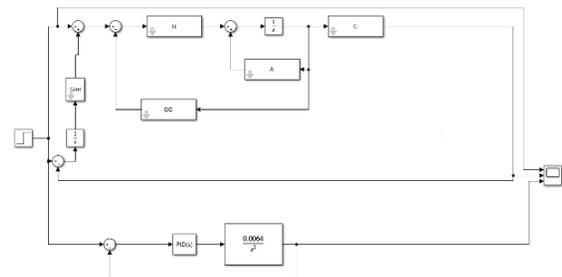


Fig. 12. Block diagrams for comparing the responses of the controllers.
 Source: Self-Implementation

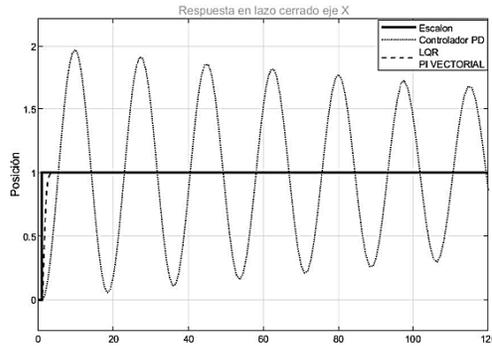


Fig. 13. Step response of the system with both controllers.
 Source: Self-Implementation

As seen in Figure 13, the line with thin dashed lines represents the response of the PD controller. It is evident that the system starts moving the ball towards the reference, but significant oscillations occur, indicating high instability in the system. These oscillations would require larger control actions and consequently more effort from the actuators, leading to potential physical problems in the future.

In contrast, the thick dashed line shows how the system reaches the reference value more quickly, indicating greater efficiency and allowing for greater durability of the mechanical couplings in the system.

5. CONCLUSIONS

A "Ball and plate" control plant allows for modeling and observing, from an academic perspective, the issues arising from nonlinearities that may occur in real processes. Additionally, it is one of the most challenging types of systems to control due to its high instability induced by significant oscillations. These oscillations manifest in its two subsystems, complicating control even further as a result of their interdependence, particularly when aiming to achieve steady-state error equal to zero.

Commonly used controllers are often inefficient for this type of plant as they lack precision, and their low effectiveness can result in permanent damage to the physical system. This issue can be overcome by using a LQR controller with integral action, which minimizes oscillations in the system and provides more precise control action, thus extending the equipment's lifespan. Therefore, the use of such controllers is proposed, especially when targeting second-order systems with sustained oscillations as control objectives.

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