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Total-Cross-Tied photovoltaic arrays with irregular configuration: a model

Arreglos fotovoltaicos con estructura cruzada con configuración irregular: un modelo

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Abstract: Photovoltaic (PV) systems are extensively used in urban environments to produce electricity. Those PV installations are designed to take profit from unused spaces such as rooftops, where many other devices are installed (antenna, chimney, etc.), thus the available space is not regular. Therefore, a mathematical model must be designed to take into account those irregularities in the PV configuration. This paper introduces a new model for those irregular PV installations, which uses an implicit formulation of the current-voltage characteristic of the modules, thus avoiding costly mathematical computations like the lambert-W function to reduce the calculation time. The model is based on an equation system formulated from the circuital relations of the total cross tied structure, which is a suitable candidate for irregular PV installations. Finally, the model is validated using a realistic PV installation based on commercial PV modules.

Keywords: Application example, calculation time reduction, implicit formulation, irregular photovoltaic array, Simulink/Matlab software.

Resumen: Los sistemas fotovoltaicos (FV) son ampliamente usados en entornos urbanos para producir electricidad. Esas instalaciones FV se diseñan para aprovechar espacios no usados como techos, donde se tienen otros dispositivos instalados (antenas, chimeneas, etc.), por lo que el espacio disponible no es regular. Por lo tanto, se debe diseñar un modelo matemático que tenga en cuenta esas irregularidades en la instalación FV. Este artículo introduce un nuevo modelo para ese tipo de instalaciones PV irregulares, el cual usa una formulación implícita de la característica corriente-voltaje de los módulos, permitiendo de esta forma evitar el uso de cálculos complejos como la función Lambert-W, lo que reduce el tiempo de cálculo. El modelo se basa en un sistemas de ecuaciones formulado a partir de las relaciones circuitales de la estructura cruzada, la cual es una candidata ideal para instalaciones FV irregulares. Finalmente, el modelo se valida usando una instalación FV realista, la cual está basada en módulos FV comerciales.

Palabras clave: Arreglo fotovoltaico irregular, ejemplo de aplicación, formulación implícita, reducción del tiempo de cálculo, software Simulink/Matlab.

1. INTRODUCTION

Photovoltaic (PV) energy is one of the renewable sources driving the worldwide power energy transition [1]. A lot of PV generators are being installed in rooftops of houses, buildings, and other structures with irregular forms or surfaces [2]. Irregular arrays are common in those surfaces where there are restrictions of installation spaces in contrast with the amount of required electric energy; for example, in rooftops of houses and buildings, it is necessary to leverage the space because the irregular shape of their rooftops or the presence of elements as chimneys, antennas, etc. See Fig. 1. Also, when Total Cross Tied (TCT) arrays are reconfigured, the optimal configuration could be an irregular array to avoid the activation of the bypass diodes (BD). To design and operate irregular arrays, PV modules are represented with mathematical models that use the lambert W function increasing the calculation time of the array [3] and therefore, reducing the power extraction during the PV array reconfiguration process. Consequently, mathematical models of irregular arrays to calculate the I-V and P-V curves reducing the processing time are required.

The objective of this paper is to propose a mathematical model to calculate the current-voltage and power-voltage curves of irregular arrays, which are commons in irregular installation spaces and after reconfigurations of a TCT arrays. The model uses an implicit formulation of the current-voltage relation of each module, avoiding the use of the lambert-W function, which increases the calculation time. With this fast and accurate model, it is possible to estimate the maximum power in the design process of PV arrays and also in its reconfiguration.

The rest of the paper is organized as follows: in Section 2, the modeling of the PV module is presented; then, the mathematical model of the irregular TCT arrays is described in Section 3, including an example to explain the proposed concept. Section 4 presents the validation of the proposed model through simulations. This section includes the mathematical development of the problem and dynamic simulations using the software PSIM. Funding and conclusions are presented in Sections 5 and 6, respectively.

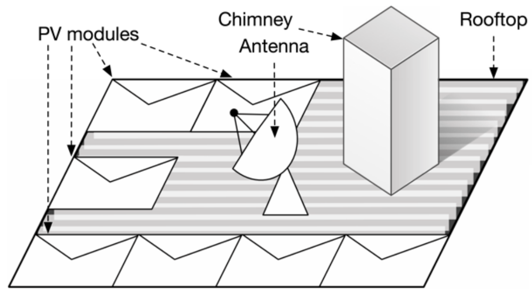

Fig. 1. *Rooftop with irregular space*

2. PV MODULE

Since PV arrays are formed by PV modules, the first step of the model is to describe, using mathematical expressions, the behavior of a single PV module. The most widely adopted representation for PV modules, which is based in an equivalent circuit, is presented in Fig. 2. Such a circuital model is named Single Diode Model (SDM), since the PN junction of the PV source is modeled with the diode D. The shunt resistor (R_h) models the leakage current and the series resistor (R_s) models the ohmic losses. In addition, the model of Fig. 2 also includes the BD.

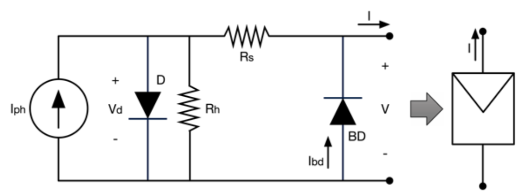

Fig. 2. *Circuital model of a PV module including the bypass diode*

The mathematical formulation of this circuital representation was analyzed in [4], which provides an implicit expression that does not require the Lambert-W function [5], thus avoiding the large computational effort required by such a specialized operation. In summary, the expression relating the module current and voltage is given in (1), where I and V are the module current and voltage, respectively.

$$f(V, I) = -I + I_{ph} - I_D - I_{Rh} + I_{BD} \qquad (1)$$

$$I_D = I_S \left[e^{\frac{V_d}{N_S \cdot \eta \cdot V_t}} - 1 \right] \quad (2)$$

$$I_{Rh} = \frac{V_d}{R_h} \quad (3)$$

$$V_d = V + (I - I_{BD}) \cdot R_s \quad (4)$$

$$I_{BD} = I_{S,BD} \left[e^{\frac{V}{\eta_{BD} \cdot V_{t,BD}}} - 1 \right] \quad (5)$$

I_{ph} is the photovoltaic current, which is almost proportional to the solar irradiance reaching the PV module. I_S, η and V_t are the inverse saturation current, the ideality factor and the thermal voltage of the PV module; N_s corresponds to the number of series-connected cells forming the module. Similarly, $I_{S,BD}$, η_{BD} and $V_{t,BD}$ are the inverse saturation current, the ideality factor and the thermal voltage of the bypass diode. Those parameters can be extracted from both the datasheet and experimental measurements using the method reported in [6].

3. IRREGULAR TCT ARRAY

The irregular TCT arrays are formed by rows with different number of modules. For example, the irregular TCT array installed in the rooftop example of Fig. 1 is electrically described in Fig. 3, which is formed by three rows: the first one has two parallel-connected modules (M_1 and M_4), the second one has a single module (M_2), and the third one has four parallel-connected modules (M_3, M_5, M_6 and M_7). The module index is assigned by numbering the modules from top-to-bottom in each column, then switching to the next column to start from the bottom again. Taking into account that all the modules of the same row have the same voltage, the row voltages are assigned by numbering the row voltages from top-to-bottom. In addition, the size of the array is described in terms of the number of rows (N) and the maximum number of columns of the array (M); hence the array in Fig. 3 has N = 3 and M = 4.

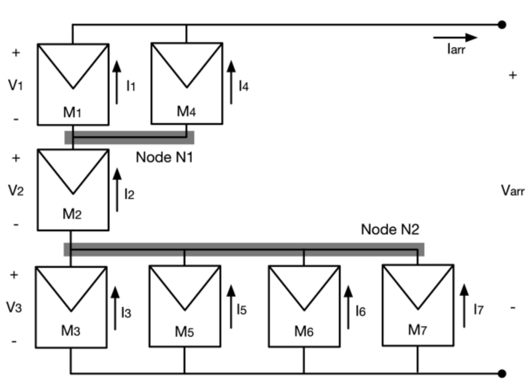

Fig. 3. *Irregular TCT array (N = 3 and M = 4)*

The parameters of the modules forming the array are described inside matrices with the nomenclature previously described, where the parameters of the missing modules are reported with a 0 value. For example, the photovoltaic current I_{ph} of all the modules in the array of Fig. 3 are reported in the following matrix:

$$M_{I_{ph}} = \begin{bmatrix} 5.5 & 4.5 & 0 & 0 \\ 2.5 & 0 & 0 & 0 \\ 5.5 & 2.5 & 4.5 & 4.5 \end{bmatrix} \quad (6)$$

In the previous matrix, the photovoltaic current of the module M_1 is 5.5 A, while the I_{ph} current of module M_6 is 4.5 A. In this way, matrix M_{Iph} provides all the photovoltaic currents of the array. The same procedure is used to describe I_S, η, V_t, N_s, R_s, R_h, $I_{S,BD}$, η_{BD} and $V_{t,BD}$ for all the modules in the array, this using the corresponding matrices: M_{Is}, M_η, M_{Vt}, M_{Ns}, M_{Rs}, M_{Rh}, $M_{Is,BD}$, $M_{\eta,BD}$, $M_{Vt,BD}$.

To analyze the PV array, it is needed to calculate the modules voltages and currents: N voltages (Vr) and N_M module currents (Im), where N_M is the number of non-zero elements from the M_{Iph} matrix. Thus, the unknown variables are given as follows:

$$V_r = [V_1 \quad V_2 \quad \cdots \quad V_N]^T \quad (7)$$

$$I_m = [I_1 \quad I_2 \quad \cdots \quad I_{N_m}]^T \quad (8)$$

Then, the system of non-linear equations describing the PV array is constructed with N_m current equations from each PV module, thus equal to (1), the sum of all the rows voltages equal to the array voltage, and N-1 Kirchhoff current equations (KCL) from each node of the PV array, i.e. the connections between rows; Fig. 4 shows the two nodes (N1 and N2) for the example TCT array. Finally, Fig. 4 shows the general structure of the equation system that must be solved to find the modules current and voltages, i.e. equations (7) and (8).

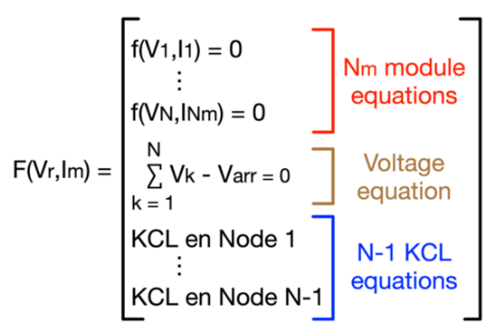

Fig. 4. *Structure of the equation system*

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The previous equation system can be solved using a numerical method or optimization algorithm, e.g. using the ***fsolve()*** function from Matlab.

4. CIRCUITAL VALIDATION

The validation of the proposed model is performed using the array example depicted in Fig. 4 (N = 3 and M = 4). This PV array has 10 unknown variables (3 voltages and 7 currents) as reported in equations (9) and (10).

$$V_r = [V_1 \quad V_2 \quad V_3]^T \tag{9}$$
$$I_m = [I_1 \quad I_2 \quad I_3 \quad I_4 \quad I_5 \quad I_6 \quad I_7]^T \tag{10}$$

Then, applying the modeling procedure reported in Section 3, the resulting equation system needed to find those unknown variables is given in Fig. 5.

$$F(V_r, I_m) = \begin{bmatrix} f(V_1, I_1) = 0 \\ f(V_2, I_2) = 0 \\ f(V_3, I_3) = 0 \\ f(V_4, I_4) = 0 \\ f(V_5, I_5) = 0 \\ f(V_6, I_6) = 0 \\ f(V_7, I_7) = 0 \\ V_1 + V_2 + V_3 - V_{arr} = 0 \\ (I_1 + I_4) - I_2 = 0 \\ I_2 - (I_3 + I_5 + I_6 + I_7) = 0 \end{bmatrix}$$

Fig. 5. *Equation system of the example TCT array (N = 3 and M = 4)*

The PV module considered for the simulations was the ERDM 85, which is formed by 36 series connected cells ($N_s = 36$) and one bypass diode. The SDM model parameters were obtained by using the procedure proposed in [6] obtaining the following values for a temperature of 25 °C and an irradiance of 1 kW/m2: $I_{ph} = 5.133$ A, η = 1.061, $I_s = 1.184$ A, $R_s = 186.4$ mΩ, and $R_h = 261.09$ Ω. Moreover, the bypass diode's parameters used for the simulations were: $\eta_{BD} = 0.269$ and $I_{S,BD} = 1.00$ μA. The temperature assumed for the cells and bypass diodes was 25 °C; hence, $V_t = V_{t,BD} = 25.7$ mV. Although the same parameters were used for all the modules (except I_{ph}), it is important to highlight that the model is the same if each module and bypass diode of the array has different parameters.

The simulations considered three scenarios represented in the matrixes M_{Iph1}, M_{Iph2} and M_{Iph3} shown below, that represent the I_{ph} parameter of the array's modules for uniform conditions (M_{Iph1}) and two different nonuniform conditions (M_{Iph2} and M_{Iph3}).

$$M_{I_{ph1}} = 5.133 \begin{bmatrix} 1.0 & 1.0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \\ 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

$$M_{I_{ph2}} = 5.133 \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}$$

$$M_{I_{ph3}} = 5.133 \begin{bmatrix} 0.25 & 0.25 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0.25 & 0.25 \end{bmatrix}$$

The equivalent circuit of the array was also implemented in Simulink/Matlab to calculate the error of the proposed model regarding the circuital implementation. The simulation results are introduced in Fig. 6, where curves obtained with the proposed model are presented in blue and the results from Simulink are presented in red with continuous, dashed, and dotted lines for M_{Iph1}, M_{Iph2}, and M_{Iph3}, respectively.

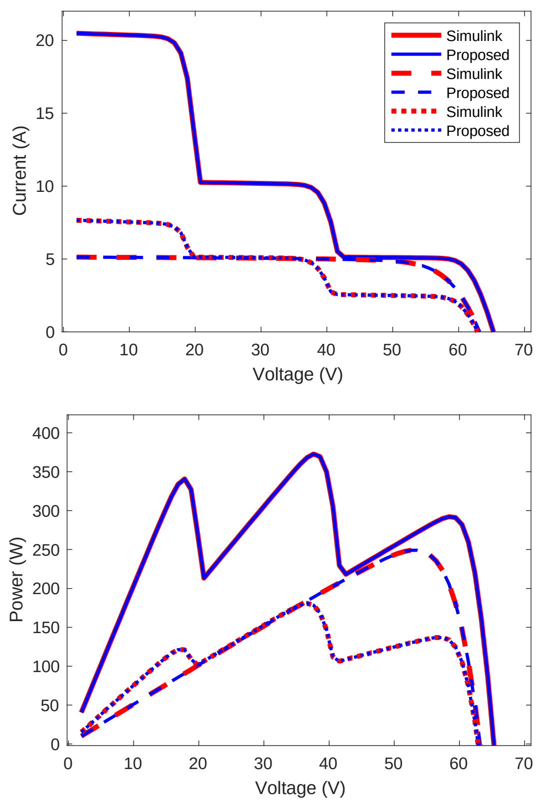

Fig. 6. *Simulation results for an irregular TCT array for three operating conditions. M_{Iph1}: continuous lines, M_{Iph2}: dashed lines, M_{Iph3}: dotted lines. Red for Simulink results and blue for the proposed model.*

For all the cases, the proposed model reproduces the circuital implementation results with normalized sum of square errors (NSSE) of 4.2e-12%, 9.3e-12%, and 24.1e-12% for M_{Iph1}, M_{Iph2}, and M_{Iph3}, respectively.

The I-V and P-V curves for M_{Iph1} indicate that an irregular TCT configuration may provide multiple maximum power points (MPPs) even if all the modules are operating under the same irradiance (i.e. uniform conditions). Moreover, the I-V curves and P-V curves for M_{Iph2} and M_{Iph3} show that irregular TCT arrays can be used to compensate nonuniform irradiance conditions. As can be observed, M_{Iph2} and M_{Iph3} have the same modules organized in two different ways; but, with the order provided by M_{Iph2}, the sum of I_{ph} in each row is the same (1.0 A); hence, the array P-V curve has only one MPP. However, with the order provided by M_{Iph3} the P-V curve has three MPPs and all of them are less than the MPP obtained with M_{Iph2}. This indicates that the proposed model can be used to implement model-based reconfiguration algorithms or other applications that require the calculation of the I-V and P-V curves of irregular TCT arrays.

5. CONCLUSIONS

This paper proposed a mathematical model that uses an implicit formulation of the current-voltage relation of each module to represent PV irregular arrays. An equation system to represent each module was derived from the circuit model; subsequently, a TCT irregular array was used to illustrate the solution procedure. Finally, the proposed model was validated through the equation system formulation of a 3x4 irregular array and its simulation using Simulink/Matlab software. The comparison of simulation results shown that the proposed model is suitable to implement model-based reconfiguration algorithms or other applications that require the calculation of the I-V and P-V curves of irregular TCT arrays, which are commonly used in rooftop of houses, buildings, and other structures.

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