

Real-Time Recording and Reconstruction of Holograms Integrating Optical and Digital Technology

Registro y Reconstrucción de Hologramas en Tiempo Real Integrando Tecnología Óptica y Digital

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Resumen

Se presenta la implementación de un interferómetro Mach-Zehnder para el registro del holograma y un procesador óptico 2F acoplado para la reconstrucción, tal que los dos procesos se realizan en tiempo real. Utilizando un sistema híbrido óptico-digital, el arreglo experimental emplea una cámara CMOS2500 y un modulador LCR2500 para lograr una grabación de alta resolución y una reconstrucción dinámica de imágenes holográficas. Los resultados experimentales demuestran la capacidad del sistema para la holografía en tiempo real. Se presentan resultados de hologramas de Fourier.

Abstract:

The implementation of a Mach-Zehnder interferometer for hologram recording and a coupled 2F optical processor for reconstruction is presented, allowing both processes to be carried out in real-time. Using a hybrid optical-digital system, the experimental setup employs a CMOS2500 camera and an LCR2500 modulator to achieve high-resolution recording and dynamic reconstruction of holographic images. The experimental results demonstrate the system's capability for real-time holography. Fourier hologram results are presented.

Palabras clave: Holografía; Interferencia; Difracción; Fourier.

Keywords: Holography; Interference; Diffraction; Fourier.

1. Introduction

Holography was invented as a method to enhance the resolution of electron microscopes. However, with the development of the laser in the 1960s, optical holography began to flourish, allowing the creation of three-dimensional images from interference patterns recorded on photosensitive media. Since its invention by Dennis Gabor in 1947 [1, 2], holography has evolved significantly [3–79] and has found applications in various fields, from science, health, engineering, art, and entertainment. This article includes the fundamental principles of holography [17, 80]. Additionally, an experimental section based on the real-time recording and reconstruction of holograms [81–83] using a hybrid optical-digital system is presented.

It is important to highlight that optical holography achieved accelerated development thanks to the invention of the laser; however, incoherent holography [66, 84, 85] has also been developing today. During the first two decades, it became one of the most attractive research topics due to its multiple applications.

Even today, interest in holography persists thanks to new materials [3, 4, 31, 36, 41, 48, 51, 57, 70–72, 83, 85, 86], optoelectronic devices, and advances in computer technology. Its historical review dates back to the mid-1960s with the development of digital computers and programming languages, which also gave rise to digital holography [33, 44, 87–95]. We can say that holography has two directions of development: the first is entirely optical, and the second is the combination of optics with the digital environment. The incorporation of digital sensors such as CMOS and CCD cameras has revolutionized holography, enabling the digital recording and processing of holograms. This has facilitated applications in metrology, biomedicine, and data storage. In 1967, Goodman and Lawrence [93] first studied digital holography. Since then, advances have been made in different fields of application, such as holographic microscopy, particle analysis, and micro-electromechanical systems, among others. The central diffraction order and double image have been problems in digital holography for in-line recordings. Numerical methods capable of addressing this problem have been discovered. Methods such as the Laplacian

operator, high-pass spectral filtering, median and subtraction operation, and four-image phase shifting have been studied and applied to improve the reconstructed hologram [33]. Currently, holographic recording sensors have high resolution, generating more data to process. Due to this, the processing time for each hologram is longer, and it is necessary to study this problem. In different works, processing time has been discussed [33, 44]. For example, in real-time microscopy, it is essential to record and reconstruct with millisecond time differences. In this case, using the GPU is favorable due to its architecture for parallel processing. Another example is the creation of holographic videos [33], where recording and reconstruction processes take too long using the CPU, making GPU usage necessary. Advances in light modulators, such as liquid crystal devices (LCR) [70, 96–100], have allowed real-time visualization and manipulation of holograms. Additionally, photorefractive materials [74, 101, 102] have played a crucial role in real-time holography. These materials can change their refractive index in response to light, allowing dynamic storage and updating of holograms. Photorefractive crystals, such as lithium niobate, barium titanate, sillenite family crystals, and certain polymers, have demonstrated unique optical properties that make them ideal for real-time holography applications. Other applications of holography include color holograms [19, 23, 69, 86, 103–107], applications in measuring deformations, vibrations, and displacements in mechanical components and structures. Holographic interferometry is a powerful tool in stress analysis and materials inspection. In biomedicine [13, 14, 28, 39, 42], digital holography has enabled high-resolution visualization and analysis of cells and tissues without the need for fluorescent markers. Holographic tomography is particularly useful for studying biological samples in three dimensions. In entertainment and art, holograms have captured the public's imagination through their use in museum exhibits, artistic installations, and the entertainment industry, including holographic concerts and live projections. Another important line is the use of holograms in document and product security, such as banknotes, credit cards, and product labels, providing an additional layer of protection against counterfeiting [73, 78].

In this article, we present an experimental study on the recording and reconstruction of Fourier holograms using a hybrid optical-digital system. The system employs a Mach-Zehnder interferometer for creating Off-Axis holograms, using a CMOS2500 camera for recording the hologram and a liquid crystal modulator (LCR2500) [77, 97] for optical reconstruction. This approach combines the advantages of digital capture and real-time visualization, offering new possibilities in holography research and practical applications.

2. Theoretical Fundamentals

A hologram is the complete recording of a wavefront, that is, its phase and amplitude. The word hologram was coined by Dennis Gabor from the words “gram” (message) and “halos” (whole). In the mid-1940s, Dennis Gabor introduced this concept in optics [2, 108], demonstrating that the phase of a wavefront, which is normally lost when captured by conventional image recording methods, can be recorded. This is because conventional recording materials have a quadratic

response, meaning they are only sensitive to the intensity of the wave. Holography is based on the principles of interference and diffraction of light. A hologram is formed when a coherent light beam (usually a laser) is split into two: an object beam that illuminates the object and a reference beam that combines with the object beam in the recording medium, creating an interference pattern that contains both amplitude and phase information. Conventional holography is optical in both recording and reconstruction. The recording is done on a photosensitive material, examples include silver halide film, photorefractive materials [75, 77, 101, 102], among others. A new line of work arises with the advent of digital technologies, leading to digital holography, which refers to an optical recording of the hologram and a digital reconstruction. In this work, we include results from our implementation, where we leverage the potential of both techniques, having built a hybrid system that allows for the recording and reconstruction process to be optical and real-time, without requiring a computational algorithm for hologram reconstruction. Our proposal also allows for parallel optical or digital processing to improve visualization quality in the reconstruction.

2.1 Hologram Recording

Holography is an optical technique that modulates both the amplitude and phase of light waves, fundamentally based on the phenomenon of interference. A hologram is the recording of the interference pattern in a physical medium (film, optical modulators, or digital medium). In a holographic optical setup, two interfering or superimposing beams are distinguished, a reference beam $U_r(x, y)$ and an object beam $U_o(x, y)$, so that the intensity $I(x, y)$ of the interference pattern recorded in the recording medium is defined as:

$$I(x, y) = |U_o(x, y) + U_r(x, y)|^2 \quad (1)$$

Expanding this expression, we obtain:

$$I(x, y) = |U_o(x, y)|^2 + |U_r(x, y)|^2 + U_o(x, y)U_r^*(x, y) + U_o^*(x, y)U_r(x, y) \quad (2)$$

where $U_r^*(x, y)$ and $U_o^*(x, y)$ are the complex conjugates of $U_r(x, y)$ and $U_o(x, y)$, respectively. The terms $|U_o(x, y)|^2$ and $|U_r(x, y)|^2$ represent the individual intensities of the object and reference waves, while the terms $U_o(x, y)U_r^*(x, y)$ and $U_o^*(x, y)U_r(x, y)$ are the cross-interference terms that contain the relative phase information between the two waves, essential for holography.

2.2 Hologram Reconstruction

To reconstruct the hologram, the recording medium is illuminated with a reading wave generally similar to the one used during recording, or simply the reference $U_r(x, y)$. The optical demodulation or reconstruction of the hologram involves the part of the reading beam diffracted from the recorded interference pattern (hologram). The diffracted field $U_d(x, y)$ can be represented as:

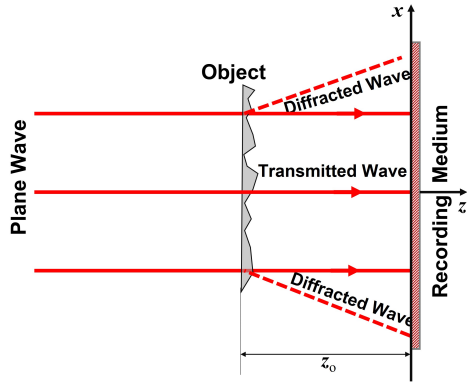
$$U_d(x, y) = I(x, y)U_r(x, y) \quad (3)$$

Substituting $I(x, y)$ in this expression, we obtain:

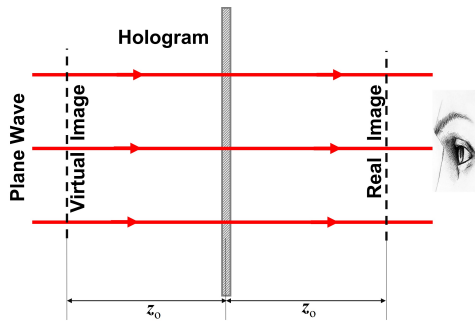
$$U_d(x, y) = [|U_o(x, y)|^2 + |U_r(x, y)|^2 + U_o(x, y)U_r^*(x, y) + U_o^*(x, y)U_r(x, y)]U_r(x, y) \quad (4)$$

Of these terms, the most important for image reconstruction is $U_o(x, y)U_r^*(x, y)$, which represents the original object wave modulated by the reference wave's intensity. This term reproduces the image of the original object when observed in the reconstructed field.

2.3 On-Axis Holograms



(a) Hologram Recording.



(b) Hologram Reconstruction.

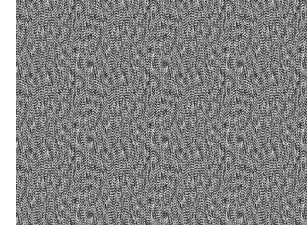
Figure 1: Schemes for On-Axis hologram recording-reconstruction. Source: Authors.

Fig. 1 presents the optical arrangement schemes for the on-axis hologram recording-reconstruction technique proposed by Gabor, known as On-Axis holograms [2, 95]. In the recording setup (Fig. 1a), the transparent object and the light source are aligned along the same axis perpendicular to the recording medium. When the object is illuminated with a collimated monochromatic beam, the light incident on the recording medium can be considered as the superposition of two waves. The first is the transmitted plane wave, also called the reference $U_r(x, y)$, whose amplitude and phase are constant over the recording medium. The light diffracted by the object is the second wave or object wave $U_o(x, y)$; on the recording medium $|U_o(x, y)| \ll |U_r(x, y)|$. The resultant amplitude at any point on the recording medium is the superposition of these two complex amplitudes, so the intensity at any point has the form of Eqs. 1-2. For simplicity, let's assume that the medium has a linear response to $I(x, y)$, so we can represent

the transmittance of the recording (hologram) as:

$$t(x, y) = t_o + \tau I(x, y), \quad (5)$$

where t_o is a constant term and τ is a parameter determined by the illumination conditions and the response of the recording medium [17, 33].



(a) Numerically calculated On-Axis hologram.



(b) Optical reconstruction of the hologram (a).

Figure 2: Example of On-Axis holograms. Source: Authors.

Fig. 1b shows the scheme for how we can optically reconstruct or visualize the object image contained in the hologram. As shown in Fig. 1b, it is illuminated with a beam of the same characteristics as the one used in the original recording. Thanks to digital technology, we can now record using a digital camera, and reconstruction can be done via a computational algorithm that simulates the optical arrangement in Fig. 1b. Whatever the reconstruction technique, the process can be represented by:

$$\begin{aligned} U_d(x, y) &= U_r(x, y)t(x, y) \\ &= U_r(x, y)(t_o + \tau U_r(x, y)^2) + \\ &\quad \tau U_r(x, y)|U_o(x, y)|^2 \\ &\quad + \tau|U_r(x, y)|^2 U_o(x, y) + \\ &\quad \tau U_r(x, y)^2 U_o^*(x, y) \end{aligned} \quad (6)$$

The first of the four terms in Eq.6 represents a uniform attenuated plane wave, corresponding to the directly transmitted beam. The second term is extremely weak compared to the other terms, considering $|U_o(x, y)| \ll |U_r(x, y)|$. The third term, except for the constant factor, is identical to the complex amplitude of the wave diffracted by the object, so this wave reconstructs a virtual image of the object at the same distance from the position of the object (z_o). The fourth term corresponds to a wavefront that resembles the original object wavefront, except it has the opposite curvature. This wave converges to form a real image of the object at the same distance z_o . It is evident that in an On-Axis hologram, in the case of optical reconstruction, an observer will see a superimposed image. The presence of this unwanted image constitutes the most significant limitation of this type of hologram. Another limitation of this holographic arrangement is the need for the object to have high average transmittance so that the second

term of Eq. 6 does not significantly interfere with the reconstructed image. In Fig.2, an example of an On-Axis hologram is presented.

2..4 Off-Axis Holograms

Emmett Leith and Juris Upatnieks [83, 88, 89, 109–113] proposed the first method for separating the virtual image from the real image (Fig. 3). The technique is based on an amplitude division interferometer, where one of the beams illuminates the object (object wave) and the second is taken as the reference wave, with the particularity that the optical axes of the two beams form an angle θ between them, as shown in Fig. 3.

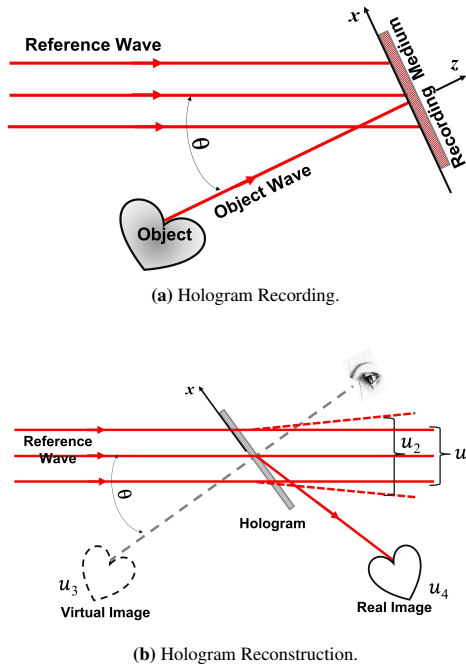


Figure 3: Schemes for Off-Axis hologram recording-reconstruction. Source: Authors.

For simplicity, suppose this reference beam is a collimated beam of uniform intensity. The complex amplitude of the wave emerging from the object at any point (x, y) of the recording medium can be written as,

$$U_o(x, y) = |U_o(x, y)|e^{-i\phi(x, y)}, \quad (7)$$

and the amplitude of the reference beam as,

$$U_r(x, y) = |U_r(x, y)|e^{i2\pi\xi_r x}, \quad \text{being } \xi_r = \frac{\sin \theta}{\lambda} \quad (8)$$

where ξ_r is the modulation frequency of the amplitude and phase of the object wave $U_o(x, y)$. Then, the resulting intensity on the recording medium is expressed by:

$$\begin{aligned} I(x, y) &= |U_r(x, y) + U_o(x, y)|^2 \\ &= |U_r(x, y)|^2 + |U_o(x, y)|^2 + \\ &|U_r||U_o|e^{-i\phi(x, y)}e^{-i2\pi\xi_r x} \\ &+ |U_r||U_o|e^{i\phi(x, y)}e^{i2\pi\xi_r x} \\ &= |U_r|^2 + |U_o|^2 + \\ &2|U_r||U_o|[\cos(\gamma_{or})] \cos(2\pi\xi_r x + \phi(x, y)) \end{aligned} \quad (9)$$

the last term of this expression shows that the amplitude and phase of the object wave are encoded as a modulation of the amplitude and phase on a spatial carrier frequency ξ_r . The factor $\cos(\gamma_{or})$ is the polarization term, with γ_{or} being the angle between the polarization planes of the object and reference waves; this parameter is important to consider because it indicates the possibility of manipulating the fringe contrast if polarized waves are used, for example, linearly. This fact will have a direct influence on the diffraction efficiency of the hologram.

Similar to the On-Axis case, the amplitude transmittance of the hologram can be written as:

$$\begin{aligned} t(x, y) &= t_0 + \tau \{ |U_o(x, y)|^2 + |U_r(x, y)|^2 \\ &+ |U_r(x, y)||U_o(x, y)|e^{-i\phi(x, y)}e^{-i2\pi\xi_r x} \\ &+ |U_r(x, y)||U_o(x, y)|e^{i\phi(x, y)}e^{i2\pi\xi_r x} \}, \end{aligned} \quad (10)$$

To reconstruct the object image, the hologram is illuminated with a plane wave of uniform amplitude, for example, with the same reference wave, as shown in Fig. 3b. Then the complex amplitude $U_d(x, y)$ is the sum of four terms, each corresponding to one of the terms in Eq. (10), and can be written as:

$$U_d(x, y) = U_r(x, y)t(x, y), \quad (11)$$

$$U_d(x, y) = U_1(x, y) + U_2(x, y) + U_3(x, y) + U_4(x, y), \quad (12)$$

where,

$$\begin{aligned} u_1(x, y) &= |U_r|e^{i2\pi\xi_r x}t_0, \\ u_2(x, y) &= |U_r|e^{i2\pi\xi_r x}\tau|U_o(x, y)|^2, \\ u_3(x, y) &= |U_r|\tau U_r^*(x, y)U_o(x, y), \\ u_4(x, y) &= |U_r|\tau U_r(x, y)U_o^*(x, y)e^{i4\pi\xi_r x} \end{aligned} \quad (13)$$

where $u_1(x, y)$ is the attenuated reference beam (plane wave transmitted directly through the hologram). This beam is surrounded by a diffraction halo due to the second term $u_2(x, y)$, whose angular propagation is determined by the object beam. $u_3(x, y)$ is the object wave except for a constant factor, and it produces a virtual image of the object; this wave forms an angle θ with the transmitted wave. $u_4(x, y)$ is the conjugate object wave that produces a real image of the object. In this case, the factor $e^{i4\pi\xi_r x}$ indicates that the conjugate wave is deflected by 2θ relative to the z axis. In conclusion, this method ensures that the two images (real and virtual) do not overlap. There is a minimum angle θ from which the real and virtual images do not overlap; this minimum value is determined by the minimum spatial frequency ξ_r , so it must be met:

$$\xi_r \geq 3\xi_m, \quad (14)$$

where ξ_m is the cutoff frequency of the object beam. This condition can be demonstrated by a Fourier transform of $U_d(x, y)$; Fig. 4 shows a pictorial description of the Fourier spectrum of Eq. (11).

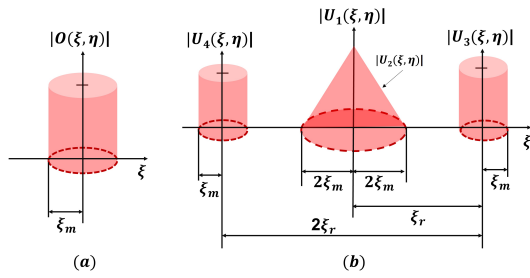
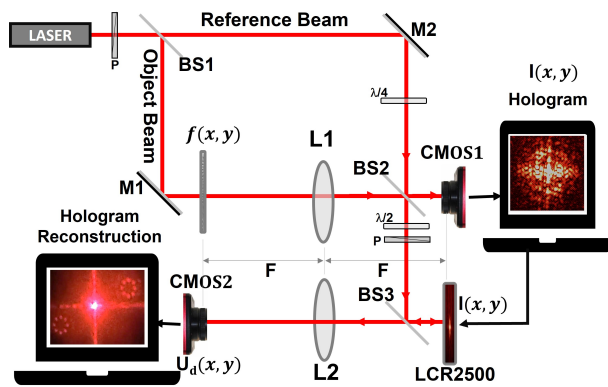


Figure 4: Scheme of the spatial frequency spectrum: (a) of the object beam, and (b) of the reconstructed off-axis hologram, identifying the location of the four terms in Eq. (13). Source: Authors.

(a). Experimental setup scheme. L1 and L2 are converging lenses with a focal length of 40 cm. M1 and M2 are flat mirrors. BS1, BS2, and BS3 are beam splitters. P are linear polarizers.



(b). Photograph of the experimental setup.



Figure 5: Experimental holographic setup for real-time recording and reconstruction. Source: Authors.

3. Optical Implementation: Real-Time Recording and Reconstruction

3..1 Description of the Hybrid Optical-Digital System

Fig. 5 shows the scheme of the experimental setup implemented to generate off-axis holograms. The part of the setup that allows hologram generation is a Mach-Zehnder interferometer. The object beam contains a 2F processor that allows the Fourier transform of the object to be obtained on the CMOS plane. This beam, in turn, is superimposed with the reference beam, thus obtaining the carrier interference pattern on which the phase and amplitude of the Fourier transform of the object are modulated, that is, the hologram is generated. In this case, the hologram is called a Fourier hologram. In parallel with the recording of the hologram, it is projected onto the LCR2500

liquid crystal spatial modulator, which is illuminated with the same reference beam to obtain the hologram reconstruction through lens L2 of the second 2F processor, and in real-time with the recording. The setup uses a solid-state laser with stable emission power at a wavelength of $\lambda = 660,5 \text{ nm}$. The primary beam was collimated to an intensity of 22 mW/cm^2 . P are cross-linear polarizers; $\lambda/4$ is a retardation plate with its axes at 45° to generate a beam with circular polarization; the $\lambda/2$ retardation plate allows adjusting the blocking of the object beam, reflected by the beam splitter BS2, such that only the reference beam transmitted by BS2 impinges on the LCR2500 modulator, where the hologram is projected, thus this beam reconstructs the hologram through L2.

3..1.1 Hologram Recording Medium

After the use of holographic film, technological advances have enabled us to have digital photosensitive media and reusable optical memories (e.g., photorefractive crystals). High spatial resolution and high temporal response digital CCD and CMOS cameras can be acquired on the market at reasonable prices. Both cameras consist of a two-dimensional array of light sensors (pixel matrix).

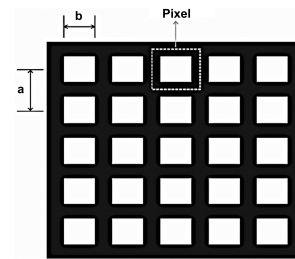


Figure 6: Sensor array scheme. Source: Authors.

Fig. 6 shows a sensor array scheme. The main difference between a CCD camera and a CMOS camera is that in the former, the signal from all pixels drives one or more amplifiers, while in the latter, each pixel has an independent amplifier. Although independent amplifiers allow the CMOS sensor to process images at a high frame rate, the properties of these amplifiers are not always uniform, resulting in slight distortion in the output image. The dark current noise of the CMOS is high. If the luminance is large enough, the CMOS image sensor can also provide high-quality images, which is one of the main characteristics that make these cameras suitable for digital holography. The sensor array size, pixel size, and fill factor (ratio of the active area $b \times b$ to the total area within a pixel $(\frac{b}{a})^2$), are the three important parameters of an image sensor. Whatever the type of camera used in holography, it is required to have a resolution such that the interference pattern can be faithfully recorded [33]. If we assume that the pixel size is Δx , then the maximum spatial frequency that the camera can resolve can be calculated using the following relation:

$$f_{\max} = \frac{2}{\lambda} \sin \left(\frac{180\lambda}{4\pi\Delta x} \right) \quad (15)$$

where λ is the wavelength of the illumination source. In our case, we use a CMOS with the following characteristics:

- Pixel size: $2,2\mu\text{m} \times 2,2\mu\text{m}$, thus $\Delta x = 2,2\mu\text{m}$

- Number of pixels: 5 MP, 2592×1944 pixels

For the wavelength of the laser source used $\lambda = 660,5$ nm, the camera can resolve fringes of up to $\sim 227 \frac{\text{Lin}}{\text{mm}}$.

3..1.2 Hologram Display Medium, LCR2500 Modulator

The behavior of a spatial light modulator is primarily characterized by two parameters: spatial resolution (number of image elements -or pixels- per unit area) and refresh rate (frequency at which the pattern displayed on the modulator can be updated). The LCR2500 screen is a spatial light modulator of the TN-LCD (Twisted Nematic Liquid Crystal Display) family [97]. It is known that liquid crystals are substances that exhibit an intermediate phase between solid and liquid states, giving them a unique combination of electrical and optical properties. These screens are key devices in the construction of hybrid interfaces, where digital information is electrically transferred to each cell and encoded in terms of local circular birefringence. The information stored in the modulator is read by light, either by transmission or reflection, as is the case with the LCR2500 used in this research, which works by reflection. The LCR2500s are easy to handle, electronically controlled, allowing digital-optical connection, high resolution, $1024(H) \times 768(V)$ with pixel size $19\mu\text{m}$, high optical efficiency, and low energy consumption.

4. Results

Fig. 7 shows two results of real-time recording and reconstruction of off-axis holograms obtained using the setup in Fig. 5. Objects **(a)** and **(d)** are binary, two-dimensional transmittance objects recorded on a plastic substrate. The results are Fourier holograms. No processing algorithms were used, neither for the holograms nor for the reconstructed images. A very good optical efficiency is observed in the reconstruction, with no perceptible delay between the recording and reconstruction processes, concluding that the system operates in real-time. This opens up the possibility for dynamic applications with organic and inorganic media, metrology, biomedical applications, among others. An interesting work perspective is the recording of holograms of three-dimensional objects. As an educational topic for teaching, it would be possible to study the registration of Fresnel and image holograms. With this setup, it is also possible to register On-Axis holograms. Phase-only holograms are possible by properly orienting the polarization state of the hologram reading beam; phase-only hologram tuning is achieved if the reading beam's polarization, being linear, is tuned such that the polarization state vibrates parallel to the LCR2500's director axis.

5. Summary and Conclusions

The article addresses the evolution and applications of holography from its invention by Dennis Gabor in 1947 to the present. Holography has progressed remarkably due to the development of the laser and digital technology. The importance of incoherent holography and the use of new materials and optoelectronic devices, such as CMOS cameras and spatial light modulators, are highlighted.

The study focuses on the recording and reconstruction of Fourier holograms using a hybrid optical-digital system. A Mach-Zehnder interferometer is employed for creating off-axis holograms, recording the holograms with a CMOS camera, and projecting them onto a liquid crystal modulator (LCR2500) for optical reconstruction in real-time.

The theoretical fundamentals of holography are described, explaining the interference and diffraction process that allows the complete information of a wavefront to be recorded. The procedures for recording and reconstructing holograms, both On-Axis and Off-Axis, are detailed, discussing the advantages and limitations of each method.

Holography has advanced significantly since its invention, finding applications in various fields such as science, medicine, engineering, art, and entertainment. Developments in laser and digital technology have been crucial for these advances. The use of a hybrid optical-digital system for recording and reconstructing Fourier holograms combines the advantages of digital capture and real-time visualization, opening new possibilities for research and practical applications.

The implemented experimental setup allows real-time recording and reconstruction of off-axis holograms, with no perceptible delay between the two processes. This is especially useful for dynamic applications in metrology and biomedicine. The hybrid system offers opportunities for teaching and researching Fresnel holograms, image holograms, and phase-only holograms, providing a versatile tool for advanced holography studies.

The incorporation of high-resolution CMOS cameras and spatial light modulators (SLM) has significantly improved the quality and efficiency of holograms, enabling their use in more complex and demanding applications. In summary, the article highlights the importance of combining optical and digital technologies to advance the field of holography, providing a solid foundation for future research and innovative applications.

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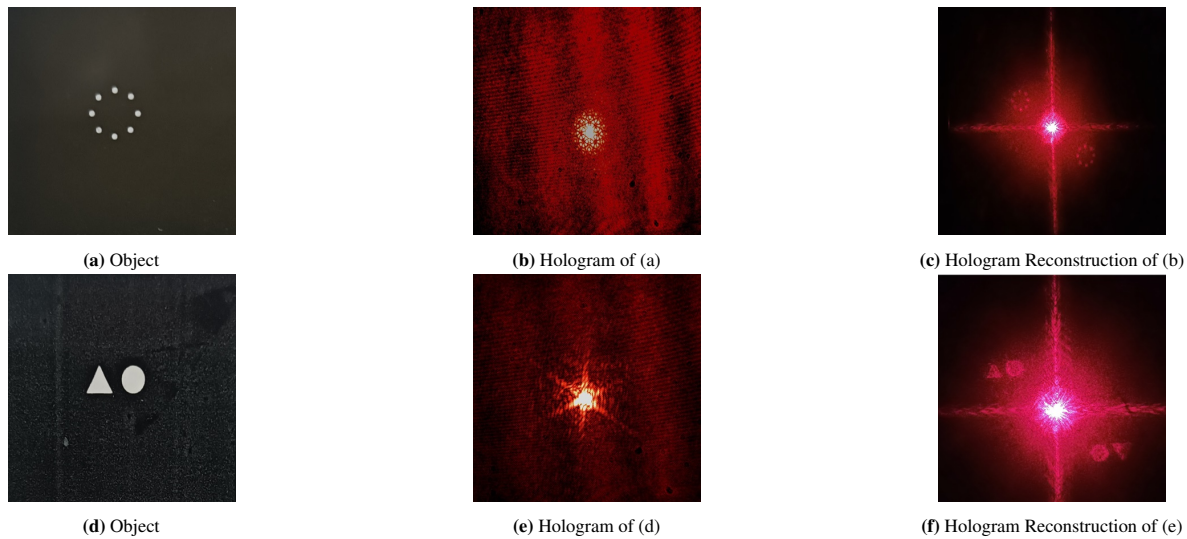


Figure 7: Off-Axis Holograms. Recording and reconstruction results obtained using the setup in Fig.5. Source: Authors.

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