**BISTUA** *Rev. FCB, Volumen 21 (2) (2023) 53-60. Pamplona-Colombia.*

BISTUA
Revista de la Facultad de Ciencias Básicas
ISSN: 0120-4211 - ISSN-e: 2711-3027

# Quadripartitioned Neutrosophic Q-Ideals of Q-Algebra

## *Q-ideales y Q-algebra neutrosófica cuadriparticionada*

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***Recibido**: Enero 6, 2023.* ***Aceptado**: Julio 04, 2023.* ***Publicado**:Julio 31, 2023.*

**Resumen**
El objetivo principal de este artículo es presentar la noción de Q-ideales (QN-Q-I) y Q-algebra (QN-Q-A) de neutrosófica cuadriparticionada, esto se deriva de una extensión de conjuntos instuitionistic difusos. Adicionalmente, formulamos algunos resultados interesantes en forma de observación de los teoremas obtenidos. Por último, para soportar los resultados obtenidos, presentamos algunos ejemplos.

***Palabras clave***: QNS; QN-Q-Q; QN-Q-I; Conjunto neutrosófico.

**Abstract**
The main aim of this paper is to procure the idea of quadripartitioned neutrosophic *Q*-ideal (in short QN-*Q*-I) of quadripartitioned neutrosophic *Q*-algebra (in short QN-*Q*-A) as an extension of intuitionistic fuzzy *Q*-ideal (in short IF-*Q*-I) of intuitionistic fuzzy *Q*-algebra (in short IF-*Q*-A). Besides, we formulate some interesting results on it in the form of remark, theorem, etc. Further, we furnish some suitable examples.

***Keywords***: QNS; QN-Q-A; QN-Q-I, Neutrosophic set.

## 1. Introduction

The idea of fuzzy set (in short F-Set) theory was first established by Zadeh [31] in 1965. In every F-Set, each element has a membership between 0 and 1. Afterwards, Atanassov [4] grounded the concept of intuitionistic fuzzy set (IF-Set) theory by generalizing the concept of F-Set, where every element has membership and non-membership values between 0 and 1. In 2005, Smarandache [29] grounded the idea of neutrosophic set (in short N-Set) by extending the idea of IF-Set. In an N-Set, every element has three independent membership values namely truth, indeterminacy and false membership values respectively. In 2013, Smarandache [30] defined and studied the notion of refined neutrosophic logic. Till now, so many researchers around the globe use N-Set in their theoretical as well as practical research [6, 9, 11, 14-17]. In 2016, Chatterjee et al. [5] presented the idea of quadripartitioned neutrosophic set (in short QNS) and established several operations on them. In 2021, Das et al. introduced the notion of topology on QNSs as an extension of neutrosophic topological space. Further, Smarandache [28-30] investigated the notion of neutro-algebra, which is the extension of partial algebra, neutro-algebraic structures and anti-algebraic structures. Follows these notions, Iseki and Tanaka [17] established the concept of *BCK*-algebra in 1978. Afterwards, Alcheikh and Sabouh [3] grounded the idea of fuzzy *BCK* algebra and fuzzy *BCK*-ideal under F-Set environment. In 2015, the idea of *BCI/BCK*-algebra under the N-Set environment was first introduced by Agboola and Davvaz [2]. Later on, Martina Jency and Arockiarani [15] grounded the idea of ideals of *BCK*-algebras under N-Set environment in 2016. Afterwards, Negger and Kim [13] studied the concept of *d*-algebra by extending the idea of BCK-algebra. Later on, the concept of *d*-ideal of *d*-algebra was presented by Negger et al. [12] in 1999. Afterwards, Jun et al. [18] extends the notion of *d*-ideal on F-Set theory, and introduced the concept of fuzzy *d*-ideal of *d*-algebra in 2000. Later on, the concept of intuitionistic fuzzy *d*-algebra was grounded by Jun et al. [20]. Thereafter, the concept of intuitionistic fuzzy *d*-ideal of *d*-algebra was studied by Hasan [28]. Hasan [29] also established the idea of intuitionistic fuzzy *d*-filter of *d*-algebra. In 2021, Das and Hassan [13] extend the concept of *d*-ideal on N-Sets, and presented the notion of neutrosophic *d*-ideal of *d*-algebra under N-Set environment. The notion of *Q*-algebra (in short

*Q*-A) was first grounded by Neggers et al. [11] in 2001. Afterwards, Abdullah and Jawad [1] introduced different types of *Q*-ideals (in short *Q*-I) in *Q*-A. Later on, Mostafa et al. [18] extends the notion of *Q*-I of *Q*-A under the F-Set environment, and presented the concept of fuzzy *Q*-ideals (in short F-*Q*-I) of fuzzy *Q*-algebra (in short F-*Q*-A). Mostafa et al. [19] further studied the intuitionistic fuzzy *Q*-ideal (in short IF-*Q*-I) of intuitionistic fuzzy *Q*-algebra (in short IF-*Q*-A) under the IF-Set environment.

The main of this paper is to procure the idea of QN-*Q*-I of QN-*Q*-A as an extension of IF-*Q*-I of IF-*Q*-A, and formulated several interesting results on it.

The remaining part of this article has been designed as follows:

In section 2, we provide some useful definitions and results on *Q*-A, F-*Q*-A, IF-*Q*-A, etc. those are relevant to the main results. In section-3, we procure the concept of QN-*Q*-I of QN-*Q*-A by generalizing the theory of IF-*Q*-I. Besides, we formulate some suitable results on QN-*Q*-I of QN-*Q*-A. In section 4, we state some future scope of research in this direction, and conclude the work done in this article.

## 2. Some relevants results

Let us consider a fixed set Ω. Then, a QNS [20] $\tilde{A}$ over Ω is defined as follows:

$\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma\in\Omega\}$,

where $T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma) \in[0, 1]$ are the degree of four independent membership namely, truth, contradiction, ignorance and falsity membership for $\gamma\in\Omega$. So, $0 \leq T_{\tilde{A}}(\gamma) + C_{\tilde{A}}(\gamma) + U_{\tilde{A}}(\gamma) + F_{\tilde{A}}(\gamma) \leq 4$.

Assume that [20] $\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma\in\Omega\}$ and $\ddot{A} = \{(\gamma, T_{\ddot{A}}(\gamma), C_{\ddot{A}}(\gamma), U_{\ddot{A}}(\gamma), F_{\ddot{A}}(\gamma)) : \gamma\in\Omega\}$ be any two QNSs over Ω. Then, $\tilde{A} \subseteq \ddot{A}$ iff $T_{\tilde{A}}(\gamma)\leq T_{\ddot{A}}(\gamma)$, $C_{\tilde{A}}(\gamma)\leq C_{\ddot{A}}(\gamma)$, $U_{\tilde{A}}(\gamma)\geq U_{\ddot{A}}(\gamma)$, $F_{\tilde{A}}(\gamma)\geq F_{\ddot{A}}(\gamma)$, for all $c\in\Omega$.

Let us consider two [20] QNSs $\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma\in\Omega\}$ and $\ddot{A} = \{(\gamma, T_{\ddot{A}}(\gamma), C_{\ddot{A}}(\gamma), U_{\ddot{A}}(\gamma), F_{\ddot{A}}(\gamma)) : \gamma\in\Omega\}$ over Ω. Then,

$\tilde{A} \cap \ddot{A} = \{(\gamma, \min\{T_{\tilde{A}}(\gamma), T_{\ddot{A}}(\gamma)\}, \min\{C_{\tilde{A}}(\gamma), C_{\ddot{A}}(\gamma)\}, \max\{U_{\tilde{A}}(\gamma), U_{\ddot{A}}(\gamma)\}, \max\{F_{\tilde{A}}(\gamma), F_{\ddot{A}}(\gamma)\}) : \gamma\in\Omega\}$;

$\tilde{A} \cup \ddot{A} = \{(\gamma, \max\{T_{\tilde{A}}(\gamma), T_{\ddot{A}}(\gamma)\}, \max\{C_{\tilde{A}}(\gamma), C_{\ddot{A}}(\gamma)\}, \min\{U_{\tilde{A}}(\gamma), U_{\ddot{A}}(\gamma)\}, \min\{F_{\tilde{A}}(\gamma), F_{\ddot{A}}(\gamma)\}) : \gamma\in\Omega\}$;

$\tilde{A}^c = \{(\gamma, F_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), T_{\tilde{A}}(\gamma)) : \gamma\in\Omega\}$.

Assume that Ω be a fixed set. Suppose that '∗' be a binary operation defined on Ω, and 0 be a constant in it. Then, the structure (Ω, ∗, 0) is said to be a *Q*-A [22] iff the following conditions hold:

(i) $\gamma * \gamma = 0, \forall\gamma \in \Omega$
(ii) $0 * \gamma = \gamma = \gamma * 0, \forall\gamma \in \Omega$
(iii) $(\gamma * \alpha) * e = (\gamma * e) * \alpha, \forall\gamma, \alpha, e \in \Omega$.

Sometime, one can refer to $\gamma \leq \alpha \Leftrightarrow \gamma * \alpha = 0$.

Let Ω be a fixed set, and $\ddot{A}$ ($\neq 0_N$) $\subseteq$ Ω. Then, $\ddot{A}$ is called a *Q*-sub-algebra (in short *Q*-S-A) [24] of a *Q*-algebra (Ω, ∗, 0), if $\gamma * \alpha \in \ddot{A}$ whenever $\gamma, \alpha \in \ddot{A}$.

A *Q*-algebra (Ω, ∗, 0) is called a commutative *Q*-algebra [24] if $\gamma * (\gamma * \alpha) = \alpha * (\alpha * \gamma), \forall\gamma, \alpha\in \Omega$, and $\alpha * (\alpha * \gamma)$ is denoted by $(\gamma \wedge \alpha)$.

A *Q*-algebra (Ω, ∗, 0) is called a bounded *Q*-algebra [24] if there exist an element $å \in \Omega$ such that $\gamma \leq å\ \forall\gamma \in \Omega$ i.e., $\gamma * å = 0, \forall\gamma \in \Omega$.

Let us consider a *Q*-algebra (Ω, ∗, 0). Then, a sub-set $\ddot{A}$ of Ω is said to be a *Q*-ideal [1] of Ω if the following conditions hold:

(i) $0 \in \ddot{A}$;
(ii) $(å * \tau) * \chi \in \ddot{A}$ and $\tau \in \ddot{A} \Rightarrow å * \chi \in \ddot{A}, \forall å, \tau, \chi \in \Omega$.

A F-Set $\ddot{A}=\{(\gamma, T_{\ddot{A}}(\gamma)) : \gamma\in\Omega\}$ over a *Q*-algebra Ω is said to be the F-*Q*-I [22] if the following inequalities hold:

(i) $T_{\ddot{A}}(0) \geq T_{\ddot{A}}(\gamma), \forall\gamma\in\Omega$;
(ii) $T_{\ddot{A}}(\gamma * \alpha) \geq \min\{T_{\ddot{A}}((\gamma * \delta) * \alpha), T_{\ddot{A}}(\delta)\}, \forall\gamma, \delta, \alpha\in\Omega$.

An IF-Set $\ddot{A}=\{(\gamma, T_{\ddot{A}}(\gamma), F_{\ddot{A}}(\gamma)) : \gamma\in\Omega\}$ over Ω is said to be an IF-*Q*-I [23] if the following conditions hold:

(i) $T_{\ddot{A}}(0) \geq T_{\ddot{A}}(\gamma), F_{\ddot{A}}(0)\leq F_{\ddot{A}}(\gamma), \forall\gamma\in\Omega$;
(ii) $T_{\ddot{A}}(\gamma * \delta) \geq \min\{T_{\ddot{A}}((\gamma * \alpha) * \delta), T_{\ddot{A}}(\alpha)\}$;
(iii) $F_{\ddot{A}}(\gamma * \delta) \leq \max\{F_{\ddot{A}}((\gamma * \alpha) * \delta), F_{\ddot{A}}(\alpha)\}$.

Let $\ddot{A}=\{(\gamma, T_{\ddot{A}}(\gamma), F_{\ddot{A}}(\gamma)) : \gamma\in\Omega\}$ be an [23] IF-*Q*-I over Ω. Then, the sets $a\text{-}T_{\ddot{A}} = \{\gamma : \gamma\in\Omega, T_{\ddot{A}}(\gamma) \geq a\}$ and $a\text{-}F_{\ddot{A}} = \{\gamma : \gamma\in\Omega, F_{\ddot{A}}(\gamma) \leq a\}$ are *Q*-ideals of Ω

## 3. Quadripartitioned Neutrosophic *Q*-Algebra and Quadripartitioned Neutrosophic *Q*-Ideal:

In this section, we present the concept of QN-*Q*-I of QN-*Q*-A, and formulate several definitions and results on QN-*Q*-I and QN-*Q*-A in the form of remark, theorem, etc.

**Definition 3.1.** Assume that Ω be a *Q*-algebra. Suppose that Ä={(γ, $T_Ä$(γ), $C_Ä$(γ), $U_Ä$(γ), $F_Ä$(γ)) : γ∈Ω} be a QNS over Ω. Then, the QNS Ä is called a QN-*Q*-A iff the following condition holds:

(i) $T_Ä(\gamma * \alpha) \geq \min\{T_Ä((\gamma * \delta) * \alpha), T_Ä(\delta)\}$;

(ii) $C_Ä(\gamma * \alpha) \geq \min\{C_Ä((\gamma * \delta) * \alpha), C_Ä(\delta)\}$;

(iii) $U_Ä(\gamma * \alpha) \leq \max\{U_Ä((\gamma * \delta) * \alpha), U_Ä(\delta)$;

(iv) $F_Ä(\gamma * \alpha) \leq \max\{F_Ä((\gamma * \delta) * \alpha), F_Ä(\delta)$, where γ, α∈ Ω.

We denote a QN-*Q*-A Ä by using the structure [(Ω, Ä), ∗, 0].

**Definition 3.2.** A QNS Ä={(γ, $T_Ä$(γ), $C_Ä$(γ), $U_Ä$(γ), $F_Ä$(γ)) : γ∈Ω} is called a quadripartitioned neutrosophic *Q*-sub-algebra (in short QN-*Q*-sub-algebra) iff the following conditions hold:

(i) $T_Ä(\gamma * \alpha) \geq \min\{T_Ä(\gamma), T_Ä(\alpha)\}$;

(ii) $C_Ä(\gamma * \alpha) \geq \min\{C_Ä(\gamma), C_Ä(\alpha)\}$;

(iii) $U_Ä(\gamma * \alpha) \leq \max\{U_Ä(\gamma), U_Ä(\alpha)\}$;

(iv) $F_Ä(\gamma * \alpha) \leq \max\{F_Ä(\gamma), F_Ä(\alpha)\}$, ∀ γ, α∈Ω.

**Definition 3.3.** A QNS Ä={(γ, $T_Ä$(γ), $C_Ä$(γ), $U_Ä$(γ), $F_Ä$(γ)) : γ∈Ω} over a *Q*-algebra Ω is called a QN-*Q*-I iff the following conditions hold:

(i) $T_Ä(0) \geq T_Ä(\gamma)$ & $T_Ä(\gamma * \alpha) \geq \min\{T_Ä((\gamma * \delta) * \alpha), T_Ä(\delta)\}$, for all γ, α, δ∈Ω;

(ii) $C_Ä(0) \geq C_Ä(\gamma)$ & $C_Ä(\gamma * \alpha) \geq \min\{C_Ä((\gamma * \delta) * \alpha), C_Ä(\delta)\}$, for all γ, α, δ∈Ω;

(iii) $U_Ä(0) \leq U_Ä(\gamma)$ & $U_Ä(\gamma * \alpha) \leq \max\{U_Ä((\gamma * \delta) * \alpha), U_Ä(\delta)\}$, for all γ, α, δ∈Ω;

(iv) $F_Ä(0) \leq F_Ä(\gamma)$ & $F_Ä(\gamma * \alpha) \leq \max\{F_Ä((\gamma * \delta) * \alpha), F_Ä(\delta)\}$, for all γ, α, δ∈Ω;

**Example 3.1.** Suppose that Ω = {0, 1, 2, 3, 4} be a fixed set. Suppose that * is a binary operation defined over Ω as follows:

| * | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 4 |
| 1 | 1 | 0 | 0 | 1 | 4 |
| 2 | 2 | 2 | 0 | 0 | 4 |
| 3 | 3 | 0 | 3 | 0 | 4 |
| 4 | 4 | 4 | 4 | 4 | 0 |

Now, we define Ä={(γ, $T_Ä$(γ), $C_Ä$(γ), $U_Ä$(γ), $F_Ä$(γ)) : γ∈Ω} as follows:

$$T_Ä = \begin{cases} 0.7. \ if\ c = 0,2,3,4 \\ 0.3, \ if\ c = 1 \end{cases}$$

$$C_Ä = \begin{cases} 0.6. \ if\ c = 0,2,3,4 \\ 0.03, \ if\ c = 1 \end{cases}$$

$$U_Ä = \begin{cases} 0.2. \ if\ c = 0,2,3,4 \\ 0.5, \ if\ c = 1 \end{cases}$$

$$F_Ä = \begin{cases} 0.05. \ if\ c = 0,2,3,4 \\ 0.68, \ if\ c = 1 \end{cases}$$

So, it is a QN-Q-sub-algebra.

Now, if we define Ä={(γ, $T_Ä$(γ), $C_Ä$(γ), $U_Ä$(γ), $F_Ä$(γ)) : γ∈Ω} as follows:

$$T_Ä = \begin{cases} 0.9. \ if\ c = 0,2 \\ 0.02, \ if\ c = 1,3,4 \end{cases}$$

$$C_Ä = \begin{cases} 0.7. \ if\ c = 0,2 \\ 0.02, \ if\ c = 1,3,4 \end{cases}$$

$$U_Ä = \begin{cases} 0.01. \ if\ c = 0,2 \\ 0.8, \ if\ c = 1,3,4 \end{cases}$$

$$F_Ä = \begin{cases} 0.04. \ if\ c = 0,2 \\ 0.97, \ if\ c = 1,3,4 \end{cases}$$

Thus, Ä is a QN-Q-I.

**Remark 3.1.** Suppose that Ä be a QN-*Q*-I of a *Q*-algebra Ω. Then, Ä is also a QN-*Q*-sub-algebra.

**Theorem 3.1.** Suppose that {$Ä_i$ : i∈Δ} be the set of some QN-*Q*-As of Ω. Then, their intersection i.e., $\cap_{i\in\Delta} Ä_i$ is also a QN-*Q*-A of Ω.

**Proof.** Suppose that {$Ä_i$ : i∈Δ} be a set of QN-*Q*-As of Ω. Clearly, $\cap_{i\in\Delta} Ä_i$ = {(α, $\wedge F_{Ä_i}(\alpha)$, $\wedge C_{Ä_i}(\alpha)$, $\vee U_{Ä_i}(\alpha)$, $\vee F_{Ä_i}(\alpha)$) : α∈Ω}.

Now,

$\wedge T_{Ä_i}(\alpha * å) = \wedge\{T_{Ä_i}(\alpha * å): i\in\Delta\}$

$\geq \wedge\{\min\{T_{Ä_i}((\alpha * \gamma) * å), T_{Ä_i}(\gamma)\}\}$

$= \min\{\wedge T_{Ä_i}((\alpha * \gamma) * å), \wedge T_{Ä_i}(\gamma)\}\}$

$\Rightarrow \wedge T_{Ä_i}(\alpha * å) \geq \min\{\wedge T_{Ä_i}((\alpha * \gamma) * å), \wedge T_{Ä_i}(\gamma)\}\}$.

Now,

$\wedge C_{Ä_i}(\alpha * å) = \wedge\{C_{Ä_i}(\alpha * å): i\in\Delta\}$

$\geq \wedge\{\min\{C_{Ä_i}((\alpha * \gamma) * å), C_{Ä_i}(\gamma)\}\}$

$= \min\{\wedge C_{\ddot{A}_i}((\alpha * \gamma) * å), \wedge C_{\ddot{A}_i}(\gamma)\}\}$

$\Rightarrow \wedge C_{\ddot{A}_i}(\alpha * å) \geq \min\{\wedge C_{\ddot{A}_i}((\alpha * \gamma) * å), \wedge C_{\ddot{A}_i}(\gamma)\}\}$.

Now,

$\vee U_{\ddot{A}_i}(\alpha * å) = \vee\{U_{\ddot{A}_i}(\alpha * å) : i \in \Delta\}$

$\geq \vee\{\min\{U_{\ddot{A}_i}((\alpha * \gamma) * å), U_{\ddot{A}_i}(\gamma)\}\}$

$= \min\{\vee U_{\ddot{A}_i}((\alpha * \gamma) * å), \vee U_{\ddot{A}_i}(\gamma)\}\}$

$\Rightarrow \vee U_{\ddot{A}_i}(\alpha * å) \geq \min\{\vee U_{\ddot{A}_i}((\alpha * \gamma) * å), \vee U_{\ddot{A}_i}(\gamma)\}\}$.

Now,

$\vee F_{\ddot{A}_i}(\alpha * å) = \vee\{F_{\ddot{A}_i}(\alpha * å) : i \in \Delta\}$

$\geq \vee\{\min\{F_{\ddot{A}_i}((\alpha * \gamma) * å), F_{\ddot{A}_i}(\gamma)\}\}$

$= \min\{\vee F_{\ddot{A}_i}((\alpha * \gamma) * å), \vee F_{\ddot{A}_i}(\gamma)\}\}$

$\Rightarrow \vee F_{\ddot{A}_i}(\alpha * å) \geq \min\{\vee F_{\ddot{A}_i}((\alpha * \gamma) * å), \vee F_{\ddot{A}_i}(\gamma)\}\}$.

Therefore, $\bigcap_{i \in \Delta} \ddot{A}_i$ is a QN-*Q*-A of Ω.

**Theorem 3.2.** Assume that $\ddot{A} = \{(\gamma, T_{\ddot{A}}(\gamma), C_{\ddot{A}}(\gamma), U_{\ddot{A}}(\gamma), F_{\ddot{A}}(\gamma)) : \gamma \in \Omega\}$ be a QN-*Q*-sub-algebra of a *Q*-algebra Ω. Then, the following holds:

(i) $T_{\ddot{A}}(0) \geq T_{\ddot{A}}(\gamma), \forall \gamma \in \Omega$;

(ii) $C_{\ddot{A}}(0) \geq C_{\ddot{A}}(\gamma), \forall \gamma \in \Omega$;

(iii) $U_{\ddot{A}}(0) \leq U_{\ddot{A}}(\gamma), \forall \gamma \in \Omega$;

(iv) $F_{\ddot{A}}(0) \leq F_{\ddot{A}}(\gamma), \forall \gamma \in \Omega$;

**Proof.** Suppose that $\ddot{A} = \{(\gamma, T_{\ddot{A}}(\gamma), C_{\ddot{A}}(\gamma), U_{\ddot{A}}(\gamma), F_{\ddot{A}}(\gamma)) : \gamma \in \Omega\}$ be a QN-*Q*-sub-algebra of a *Q*-algebra Ω. Hence, $T_{\ddot{A}}(\gamma * \delta) \geq \min\{T_{\ddot{A}}(\gamma), T_{\ddot{A}}(\delta)\}$, $C_{\ddot{A}}(\gamma * \delta) \geq \min\{C_{\ddot{A}}(\gamma), C_{\ddot{A}}(\delta)\}$, $U_{\ddot{A}}(\gamma * \delta) \leq \max\{U_{\ddot{A}}(\gamma), U_{\ddot{A}}(\delta)\}$, $F_{\ddot{A}}(\gamma * \delta) \leq \max\{F_{\ddot{A}}(\gamma), F_{\ddot{A}}(\delta)\}$, $\forall \gamma, \delta \in \Omega$.

Now, we have

$T_{\ddot{A}}(0) = T_{\ddot{A}}(\gamma * \gamma)$

$\geq \min\{T_{\ddot{A}}(\gamma), T_{\ddot{A}}(\gamma)\}$

$= T_{\ddot{A}}(\gamma)$

$\Rightarrow T_{\ddot{A}}(0) \geq T_{\ddot{A}}(\gamma), \forall \gamma \in \Omega$.

$C_{\ddot{A}}(0) = C_{\ddot{A}}(\gamma * \gamma)$

$\geq \min\{C_{\ddot{A}}(\gamma), C_{\ddot{A}}(\gamma)\}$

$= C_{\ddot{A}}(\gamma)$

$\Rightarrow C_{\ddot{A}}(0) \geq C_{\ddot{A}}(\gamma), \forall \gamma \in \Omega$.

$U_{\ddot{A}}(0) = U_{\ddot{A}}(\gamma * \gamma)$

$\leq \max\{U_{\ddot{A}}(\gamma), U_{\ddot{A}}(\gamma)\}$

$= U_{\ddot{A}}(\gamma)$

$\Rightarrow U_{\ddot{A}}(0) \leq U_{\ddot{A}}(\gamma), \forall \gamma \in \Omega$.

$F_{\ddot{A}}(0) = F_{\ddot{A}}(\gamma * \gamma)$

$\leq \max\{F_{\ddot{A}}(\gamma), F_{\ddot{A}}(\gamma)\}$

$= F_{\ddot{A}}(\gamma)$

$\Rightarrow F_{\ddot{A}}(0) \leq F_{\ddot{A}}(\gamma), \forall \gamma \in \Omega$.

**Theorem 3.3.** If $\{\tilde{A}_i : i \in \Delta\}$ be a set of QN-*Q*-I's of a *Q*-algebra Ω. Then, their intersection i.e., $\bigcap_{i \in \Delta} \tilde{A}_i$ is also a QN-*Q*-I of Ω.

**Proof.** Assume that $\{\tilde{A}_i : i \in \Delta\}$ be the set of PN-*Q*-I's of a *Q*-algebra Ω. Therefore,

(i) $T_{\tilde{A}_i}(0) \geq T_{\tilde{A}_i}(\delta)$ & $T_{\tilde{A}_i}(\delta * \gamma) \geq \min\{T_{\tilde{A}_i}((\delta * \alpha) * \gamma), T_{\tilde{A}_i}(\alpha)\}, \forall \delta, \gamma, \alpha \in \Omega$ and $i \in \Delta$;

(ii) $C_{\tilde{A}_i}(0) \geq C_{\tilde{A}_i}(\delta)$ & $C_{\tilde{A}_i}(\delta * \gamma) \geq \min\{C_{\tilde{A}_i}((\delta * \alpha) * \gamma), C_{\tilde{A}_i}(\alpha)\}, \forall \delta, \gamma, \alpha \in \Omega$ and $i \in \Delta$;

(iii) $U_{\tilde{A}_i}(0) \leq U_{\tilde{A}_i}(\delta)$ & $U_{\tilde{A}_i}(\delta * \gamma) \leq \max\{U_{\tilde{A}_i}((\delta * \alpha) * \gamma), U_{\tilde{A}_i}(\alpha)\}, \forall \delta, \gamma, \alpha \in \Omega$ and $i \in \Delta$;

(iv) $F_{\tilde{A}_i}(0) \leq F_{\tilde{A}_i}(\delta)$ & $F_{\tilde{A}_i}(\delta * \gamma) \leq \max\{F_{\tilde{A}_i}((\delta * \alpha) * \gamma), F_{\tilde{A}_i}(\alpha)\}, \forall \delta, \gamma, \alpha \in \Omega$ and $i \in \Delta$.

Clearly, $\bigcap_{i \in \Delta} \tilde{A}_i = \{(\delta, \wedge T_{\tilde{A}_i}(\delta), \wedge C_{\tilde{A}_i}(\delta), \vee U_{\tilde{A}_i}(\delta), \vee F_{\tilde{A}_i}(\delta)) : \delta \in \Omega\}$.

Now, we have

$T_{\tilde{A}_i}(0) \geq T_{\tilde{A}_i}(\delta), \forall \delta \in \Omega$ and $i \in \Delta$

$\Rightarrow \wedge T_{\tilde{A}_i}(0) \geq \wedge T_{\tilde{A}_i}(\delta)$.

$C_{\tilde{A}_i}(0) \geq C_{\tilde{A}_i}(\delta), \forall \delta \in \Omega$ and $i \in \Delta$

$\Rightarrow \wedge C_{\tilde{A}_i}(0) \geq \wedge C_{\tilde{A}_i}(\delta)$.

$U_{\tilde{A}_i}(0) \leq U_{\tilde{A}_i}(\delta), \forall \delta \in \Omega$ and $i \in \Delta$

$\Rightarrow \vee U_{\tilde{A}_i}(0) \leq \vee U_{\tilde{A}_i}(\delta)$.

$F_{\tilde{A}_i}(0) \leq F_{\tilde{A}_i}(\delta), \forall \delta \in \Omega$ and $i \in \Delta$

$\Rightarrow \vee F_{\tilde{A}_i}(0) \leq \vee F_{\tilde{A}_i}(\delta)$.

Further, we have

$T_{\tilde{A}_i}(\delta * \gamma) \geq \min\{T_{\tilde{A}_i}((\delta * \alpha) * \gamma), T_{\tilde{A}_i}(\alpha)\}, \forall \delta, \gamma, \alpha \in \Omega$ and $i \in \Delta$.

$\Rightarrow \wedge T_{\tilde{A}_i}(\delta * \gamma) \geq \wedge \min\{T_{\tilde{A}_i}((\delta * \alpha) * \gamma), T_{\tilde{A}_i}(\alpha)\}$

$= \min\{\wedge T_{\tilde{A}_i}((\delta * \alpha) * \gamma), \wedge T_{\tilde{A}_i}(\alpha)\}$

$\Rightarrow \wedge T_{\tilde{A}_i}(\delta * \gamma) \geq \min\{\wedge T_{\tilde{A}_i}((\delta * \alpha) * \gamma), \wedge T_{\tilde{A}_i}(\alpha)\}$.

$C_{\tilde{A}_i}(\delta * \gamma) \geq \min\{C_{\tilde{A}_i}((\delta * \alpha) * \gamma), C_{\tilde{A}_i}(\alpha)\}, \forall \gamma, \alpha \in \Omega$ and $i \in \Delta$.

$\Rightarrow \wedge C_{\tilde{A}_i}(\delta * \gamma) \geq \wedge \min\{C_{\tilde{A}_i}((\delta * \alpha) * \gamma), C_{\tilde{A}_i}(\alpha)\}$

$= \min\{\wedge C_{\tilde{A}_i}((\delta * \alpha) * \gamma), \wedge C_{\tilde{A}_i}(\alpha)\}$

$\Rightarrow \wedge C_{\tilde{A}_i}(\delta * \gamma) \geq \min\{\wedge C_{\tilde{A}_i}((\delta * \alpha) * \gamma), \wedge C_{\tilde{A}_i}(\alpha)\}$.

$U_{\tilde{A}_i}(\delta * \gamma) \leq \max\{U_{\tilde{A}_i}((\delta * \alpha) * \gamma), U_{\tilde{A}_i}(\alpha)\}, \forall \delta, \gamma, \alpha \in \Omega$ and $i \in \Delta$.

$\Rightarrow \vee U_{\tilde{A}_i}(\delta * \gamma) \leq \vee \max\{U_{\tilde{A}_i}((\delta * \alpha) * \gamma), U_{\tilde{A}_i}(\alpha)\}$

$= \max \{\vee U_{\tilde{A}_i}((\delta * \alpha) * \gamma), \vee U_{\tilde{A}_i}(\alpha)\}$

$\Rightarrow \vee U_{\tilde{A}_i}(\delta * \gamma) \leq \max \{\vee U_{\tilde{A}_i}((\delta * \alpha) * \gamma), \vee U_{\tilde{A}_i}(\alpha)\}$.

$F_{\tilde{A}_i}(\delta * \gamma) \leq \max \{F_{\tilde{A}_i}((\delta * \alpha) * \gamma), F_{\tilde{A}_i}(\alpha)\}, \forall \delta, \gamma, \alpha \in \Omega$ and $i \in \Delta$.

$\Rightarrow \vee F_{\tilde{A}_i}(\delta * \gamma) \leq \vee \max \{F_{\tilde{A}_i}((\delta * \alpha) * \gamma), F_{\tilde{A}_i}(\alpha)\}$

$= \max \{\vee F_{\tilde{A}_i}((\delta * \alpha) * \gamma), \vee F_{\tilde{A}_i}(\alpha)\}$

$\Rightarrow \vee F_{\tilde{A}_i}(\delta * \gamma) \leq \max \{\vee F_{\tilde{A}_i}((\delta * \alpha) * \gamma), \vee F_{\tilde{A}_i}(\alpha)\}$.

Therefore, $\bigcap_{i \in \Delta} \tilde{A}_i$ is a QN-*Q*-I of *Q*-algebra Ω.

**Corollary 3.1.** Suppose that $\tilde{A} = \{(\alpha, T_{\tilde{A}}(\alpha), C_{\tilde{A}}(\alpha), U_{\tilde{A}}(\alpha), F_{\tilde{A}}(\alpha)) : \alpha \in \Omega\}$ be a QN-*Q*-I of a *Q*-algebra Ω. Then, the QNS $\tilde{A}$ is also a quadripartitioned neutrosophic BCK-ideal of the BCK-algebra Ω.

**Theorem 3.4.** Suppose that $\tilde{A} = \{(\alpha, T_{\tilde{A}}(\alpha), C_{\tilde{A}}(\alpha), U_{\tilde{A}}(\alpha), F_{\tilde{A}}(\alpha)) : \alpha \in \Omega\}$ be a QN-*Q*-I over a *Q*-algebra Ω. If $\alpha, \gamma, \delta \in \Omega$ such that $\alpha * \gamma \leq \delta$, then $T_{\tilde{A}}(\alpha) \geq \min \{T_{\tilde{A}}(\gamma), T_{\tilde{A}}(\delta)\}$, $C_{\tilde{A}}(\alpha) \geq \min \{C_{\tilde{A}}(\gamma), C_{\tilde{A}}(\delta)\}$, $U_{\tilde{A}}(\alpha) \leq \max \{U_{\tilde{A}}(\gamma), U_{\tilde{A}}(\delta)\}$ and $F_{\tilde{A}}(\alpha) \leq \max \{F_{\tilde{A}}(\gamma), F_{\tilde{A}}(\delta)\}$.

**Proof.** Let $\tilde{A} = \{(\alpha, T_{\tilde{A}}(\alpha), C_{\tilde{A}}(\alpha), U_{\tilde{A}}(\alpha), F_{\tilde{A}}(\alpha)): \alpha \in \Omega\}$ be a QN-*Q*-ideal over a *Q*-algebra Ω. Suppose that $\alpha, \gamma, \delta \in \Omega$ such that $\alpha * \gamma \leq \delta$. Therefore, $(\alpha * \gamma) * \delta = 0$.

Now, we have

$T_{\tilde{A}}(\alpha) = T_{\tilde{A}}(\alpha * 0) \geq \min \{T_{\tilde{A}}((\alpha * \gamma) * 0), T_{\tilde{A}}(\gamma)\}$

$= \min \{T_{\tilde{A}}((\alpha * \gamma)), T_{\tilde{A}}(\gamma)\}$

$\geq \min \{\min \{T_{\tilde{A}}((\alpha * \delta) * \gamma), T_{\tilde{A}}(\delta)\}, T_{\tilde{A}}(\gamma)\}$

$= \min \{T_{\tilde{A}}((\alpha * \gamma) * \delta), T_{\tilde{A}}(\delta), T_{\tilde{A}}(\gamma)\}$

$= \min \{T_{\tilde{A}}(0), T_{\tilde{A}}(\delta), T_{\tilde{A}}(\gamma)\}$

$= \min \{T_{\tilde{A}}(\gamma), T_{\tilde{A}}(\delta)\}$

$\Rightarrow T_{\tilde{A}}(\alpha) \geq \min \{T_{\tilde{A}}(\gamma), T_{\tilde{A}}(\delta)\}$.

$C_{\tilde{A}}(\alpha) = C_{\tilde{A}}(\alpha * 0) \geq \min \{C_{\tilde{A}}((\alpha * \gamma) * 0), C_{\tilde{A}}(\gamma)\}$

$= \min \{C_{\tilde{A}}((\alpha * \gamma)), C_{\tilde{A}}(\gamma)\}$

$\geq \min \{\min \{C_{\tilde{A}}((\alpha * \delta) * \gamma), C_{\tilde{A}}(\delta)\}, C_{\tilde{A}}(\gamma)\}$

$= \min \{C_{\tilde{A}}((\alpha * \gamma) * \delta), C_{\tilde{A}}(\delta), C_{\tilde{A}}(\gamma)\}$

$= \min \{C_{\tilde{A}}(0), C_{\tilde{A}}(\delta), C_{\tilde{A}}(\gamma)\}$

$= \min \{C_{\tilde{A}}(\gamma), C_{\tilde{A}}(\delta)\}$

$\Rightarrow C_{\tilde{A}}(\alpha) \geq \min \{C_{\tilde{A}}(\gamma), C_{\tilde{A}}(\delta)\}$.

$U_{\tilde{A}}(\alpha) = U_{\tilde{A}}(\alpha * 0) \leq \max \{U_{\tilde{A}}((\alpha * \gamma) * 0), U_{\tilde{A}}(\gamma)\}$

$= \max \{ U_{\tilde{A}}((\alpha * \gamma)), U_{\tilde{A}}(\gamma)\}$

$\leq \max \{\max \{U_{\tilde{A}}((\alpha * \delta) * \gamma), U_{\tilde{A}}(\delta)\}, U_{\tilde{A}}(\gamma)\}$

$= \max \{U_{\tilde{A}}((\alpha * \gamma) * \delta), U_{\tilde{A}}(\delta), U_{\tilde{A}}(\gamma)\}$

$= \max \{U_{\tilde{A}}(0), U_{\tilde{A}}(\delta), U_{\tilde{A}}(\gamma)\}$

$= \max\{ U_{\tilde{A}}(\gamma), U_{\tilde{A}}(\delta)\}$

$\Rightarrow U_{\tilde{A}}(\alpha) \leq \max\{ U_{\tilde{A}}(\gamma), U_{\tilde{A}}(\delta)\}$.

Further, we have

$F_{\tilde{A}}(\alpha) = F_{\tilde{A}}(\alpha * 0) \leq \max \{F_{\tilde{A}}((\alpha * \gamma) * 0), F_{\tilde{A}}(\gamma)\}$

$= \max \{F_{\tilde{A}}((\alpha * \gamma)), F_{\tilde{A}}(\gamma)\}$

$\leq \max \{\max \{F_{\tilde{A}}((\alpha * \delta) * \gamma), F_{\tilde{A}}(\delta)\}, F_{\tilde{A}}(\gamma)\}$

$= \max \{F_{\tilde{A}}((\alpha * \gamma) * \delta), F_{\tilde{A}}(\delta), F_{\tilde{A}}(\gamma)\}$

$= \max \{F_{\tilde{A}}(0), F_{\tilde{A}}(\delta), F_{\tilde{A}}(\gamma)\}$

$= \max \{F_{\tilde{A}}(\gamma), F_{\tilde{A}}(\delta)\}$

$\Rightarrow F_{\tilde{A}}(\alpha) \leq \max \{F_{\tilde{A}}(\gamma), F_{\tilde{A}}(\delta)\}$.

**Theorem 3.5.** Let $\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma \in \Omega\}$ be a QN-*Q*-I over a *Q*-algebra Ω. If $\gamma, h \in \Omega$ such that $\gamma \leq h$, then $T_{\tilde{A}}(\gamma) \geq T_{\tilde{A}}(h)$, $C_{\tilde{A}}(\gamma) \geq C_{\tilde{A}}(h)$, $U_{\tilde{A}}(\gamma) \leq U_{\tilde{A}}(h)$ and $F_{\tilde{A}}(\gamma) \leq F_{\tilde{A}}(h)$.

**Proof.** Assume that $\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma \in \Omega\}$ be a QN-*Q*-I over a *Q*-algebra Ω. Suppose that, γ and α be two elements of Ω such that $\gamma \leq \alpha$. Therefore, $\gamma * \alpha = 0$.

Now, we have

$T_{\tilde{A}}(\gamma) = T_{\tilde{A}}(\gamma * 0) \geq \min \{T_{\tilde{A}}((\gamma * \alpha) * 0), T_{\tilde{A}}(\alpha)\}$

$= \min \{T_{\tilde{A}}((\gamma * \alpha)), T_{\tilde{A}}(\alpha)\}$

$= \min \{T_{\tilde{A}}(0), T_{\tilde{A}}(\alpha)\}$

$= T_{\tilde{A}}(\alpha)$

$\Rightarrow T_{\tilde{A}}(\gamma) \geq T_{\tilde{A}}(\alpha)$.

$C_{\tilde{A}}(\gamma) = C_{\tilde{A}}(\gamma * 0) \geq \min \{C_{\tilde{A}}((\gamma * \alpha) * 0), C_{\tilde{A}}(\alpha)\}$

$= \min \{C_{\tilde{A}}((\gamma * \alpha)), C_{\tilde{A}}(\alpha)\}$

$= \min \{C_{\tilde{A}}(0), C_{\tilde{A}}(\alpha)\}$

$= C_{\tilde{A}}(\alpha)$

$\Rightarrow C_{\tilde{A}}(\gamma) \geq C_{\tilde{A}}(\alpha)$.

$U_{\tilde{A}}(\gamma) = U_{\tilde{A}}(\gamma * 0) \leq \max \{U_{\tilde{A}}((\gamma * \alpha) * 0), U_{\tilde{A}}(\alpha)\}$

$= \max \{U_{\tilde{A}}((\gamma * \alpha)), U_{\tilde{A}}(\alpha)\}$

$= \max \{U_{\tilde{A}}(0), U_{\tilde{A}}(\alpha)\}$

$= U_{\tilde{A}}(\alpha)$

$\Rightarrow U_{\tilde{A}}(\gamma) \leq U_{\tilde{A}}(\alpha)$.

$F_{\tilde{A}}(\gamma) = F_{\tilde{A}}(\gamma * 0) \leq \max \{F_{\tilde{A}}((\gamma * \alpha) * 0), F_{\tilde{A}}(\alpha)\}$

$= \max \{F_{\tilde{A}}((\gamma * \alpha)), F_{\tilde{A}}(\alpha)\}$

$= \max \{F_{\tilde{A}}(0), F_{\tilde{A}}(\alpha)\}$

$= F_{\tilde{A}}(\alpha)$

$\Rightarrow F_{\tilde{A}}(\gamma) \leq F_{\tilde{A}}(\alpha)$.

**Theorem 3.6.** If $\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma \in \Omega\}$ be a QN-*Q*-sub-algebra over a *Q*-algebra $\Omega$, then the sets a-$T_{\tilde{A}}$= $\{\gamma: \gamma \in \Omega, T_{\tilde{A}}(\gamma) \geq a\}$, a-$C_{\tilde{A}}$ = $\{\gamma: \gamma \in \Omega, C_{\tilde{A}}(\gamma) \geq a\}$, a-$U_{\tilde{A}}$ = $\{\gamma: \gamma \in \Omega, U_{\tilde{A}}(\gamma) \leq a\}$ and a-$F_{\tilde{A}}$ = $\{\gamma: \gamma \in \Omega, F_{\tilde{A}}(\gamma) \leq a\}$ are the *Q*-sub-algebra of $\Omega$.

**Proof.** Let $\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma \in \Omega\}$ be a QN-*Q*-sub-algebra over a *Q*-algebra $\Omega$. Therefore,

(i) $T_{\tilde{A}}(\gamma * \alpha) \geq \min \{T_{\tilde{A}}(\gamma), T_{\tilde{A}}(\alpha)\}$;

(ii) $C_{\tilde{A}}(\gamma * \alpha) \geq \min \{C_{\tilde{A}}(\gamma), C_{\tilde{A}}(\alpha)\}$;

(iii) $U_{\tilde{A}}(\gamma * \alpha) \leq \max \{U_{\tilde{A}}(\gamma), U_{\tilde{A}}(\alpha)\}$;

(iv) $F_{\tilde{A}}(\gamma * \alpha) \leq \max \{F_{\tilde{A}}(\gamma), F_{\tilde{A}}(\alpha)\}$, where $\gamma, \alpha \in \Omega$.

Assume that, $\gamma, \alpha \in$a-$\hat{A}_Y$. This implies, $T_{\tilde{A}}(\gamma) \geq a$, $T_{\tilde{A}}(\alpha) \geq a$.

Therefore, $T_{\tilde{A}}(\gamma * \alpha) \geq \min \{T_{\tilde{A}}(\gamma), T_{\tilde{A}}(\alpha)\} \geq \min\{a, a\} \geq a$.

Hence, a-$T_{\tilde{A}}$ = $\{\gamma: \gamma \in \Omega, T_{\tilde{A}}(\gamma) \geq a\}$ is a *Q*-sub-algebra of $\Omega$.

Assume that, $\gamma, \alpha \in$a-$C_{\tilde{A}}$. This implies, $C_{\tilde{A}}(\gamma) \geq a$, $C_{\tilde{A}}(\alpha) \geq a$.

Therefore, $C_{\tilde{A}}(\gamma * \alpha) \geq \min \{C_{\tilde{A}}(\gamma), C_{\tilde{A}}(\alpha)\} \geq \min \{a, a\} \geq a$.

Hence, a-$C_{\tilde{A}}$ = $\{\gamma: \gamma \in \Omega, C_{\tilde{A}}(\gamma) \geq a\}$ is a *Q*-sub-algebra of $\Omega$.

Assume that, $\gamma, \alpha \in$a-$U_{\tilde{A}}$. This implies, $U_{\tilde{A}}(\gamma) \leq a$, $U_{\tilde{A}}(\alpha) \leq a$.

Therefore, $U_{\tilde{A}}(\gamma * \alpha) \leq \max \{U_{\tilde{A}}(\gamma), U_{\tilde{A}}(\alpha)\} \leq \max \{a, a\} \leq a$.

Hence, a-$U_{\tilde{A}}$ = $\{\gamma: \gamma \in \Omega, U_{\tilde{A}}(\gamma) \leq a\}$ is a *Q*-sub-algebra of $\Omega$.

Assume that, $\gamma, \alpha \in$a-$F_{\tilde{A}}$. This implies, $F_{\tilde{A}}(\gamma) \leq a$, $F_{\tilde{A}}(\alpha) \leq a$.

Therefore, $F_{\tilde{A}}(\gamma * \alpha) \leq \max \{F_{\tilde{A}}(\gamma), F_{\tilde{A}}(\alpha)\} \leq \max \{a, a\} \leq a$.

Hence, a-$F_{\tilde{A}}$ = $\{\gamma: \gamma \in \Omega, F_{\tilde{A}}(\gamma) \leq a\}$ is a *Q*-sub-algebra of $\Omega$.

**Theorem 3.7.** Let $\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma \in \Omega\}$ be a QN-*Q*-I of $\Omega$, then the sets $\Omega(T)=\{\gamma \in \Omega: T_{\tilde{A}}(\gamma)=T_{\tilde{A}}(0)\}$, $\Omega(C)=\{\gamma \in \Omega: C_{\tilde{A}}(\gamma)=C_{\tilde{A}}(0)\}$, $\Omega(U)=\{\gamma \in \Omega: U_{\tilde{A}}(\gamma)= U_{\tilde{A}}(0)\}$ and $\Omega(F)=\{\gamma \in \Omega: F_{\tilde{A}}(\gamma)= F_{\tilde{A}}(0)\}$ are *Q*-ideals of $\Omega$.

**Proof.** Assume that $\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma \in \Omega\}$ be a QN-*Q*-I of $\Omega$. Therefore,

(i) $T_{\tilde{A}}(0) \geq T_{\tilde{A}}(\gamma)$ & $T_{\tilde{A}}(\gamma * \alpha) \geq \min\{T_{\tilde{A}}((\gamma * \delta) * \alpha), T_{\tilde{A}}(\delta)\}, \forall \gamma, \alpha, \delta \in \Omega$;

(ii) $C_{\tilde{A}}(0) \geq C_{\tilde{A}}(\gamma)$ & $C_{\tilde{A}}(\gamma * \alpha) \geq \min\{C_{\tilde{A}}((\gamma * \delta) * \alpha), C_{\tilde{A}}(\delta)\}, \forall \gamma, \alpha, \delta \in \Omega$;

(iii) $U_{\tilde{A}}(0) \leq U_{\tilde{A}}(\gamma)$ & $U_{\tilde{A}}(\gamma * \alpha) \leq \max \{U_{\tilde{A}}((\gamma * \delta) * \alpha), U_{\tilde{A}}(\delta)\}, \forall \gamma, \alpha, \delta \in \Omega$;

(iv) $F_{\tilde{A}}(0) \leq F_{\tilde{A}}(\gamma)$ & $F_{\tilde{A}}(\gamma * \alpha) \leq \max \{F_{\tilde{A}}((\gamma * \delta) * \alpha), F_{\tilde{A}}(\delta)\}$, for all $\gamma, \alpha, \delta \in \Omega$.

Since, $T_{\tilde{A}}(0)=T_{\tilde{A}}(0)$, $C_{\tilde{A}}(0)=C_{\tilde{A}}(0)$, $U_{\tilde{A}}(0)=U_{\tilde{A}}(0)$ and $F_{\tilde{A}}(0)=F_{\tilde{A}}(0)$, so $0 \in \Omega(T)$, $0 \in \Omega(C)$, $0 \in \Omega(U)$ and $0 \in \Omega(F)$.

Let $(\delta * \alpha) * e \in \Omega(T)$ and $\alpha \in \Omega(T)$. Therefore, $T_{\tilde{A}}((\delta * \alpha) * e)=T_{\tilde{A}}(0)$ and $T_{\tilde{A}}(\alpha)=T_{\tilde{A}}(0)$.

Clearly, $T_{\tilde{A}}(0) \geq T_{\tilde{A}}(\delta * e)$ (1)

Now, we have

$T_{\tilde{A}}(\delta * e) \geq \min\{T_{\tilde{A}}((\delta * \alpha) * e), T_{\tilde{A}}(\alpha)\} = \min \{T_{\tilde{A}}(0), T_{\tilde{A}}(0)\} = T_{\tilde{A}}(0)$

$\Rightarrow T_{\tilde{A}}(\delta * e) \geq T_{\tilde{A}}(0)$ (2)

From eq. (1) & eq. (2), we get $T_{\tilde{A}}(\delta * e) = T_{\tilde{A}}(0)$, which implies, $\delta * e \in \Omega(T)$. Hence, $\Omega(T)=\{\gamma\in\Omega: T_{\tilde{A}}(\gamma)=T_{\tilde{A}}(0)\}$ is a *Q*-ideal of Ω.

Similarly, it can be shown that, the sets $\Omega(C)=\{\gamma\in\Omega: C_{\tilde{A}}(\gamma)=C_{\tilde{A}}(0)\}$, $\Omega(U)=\{\gamma\in\Omega: U_{\tilde{A}}(\gamma)=U_{\tilde{A}}(0)\}$ and $\Omega(F)=\{\gamma\in\Omega: F_{\tilde{A}}(\gamma)=F_{\tilde{A}}(0)\}$ are *Q*-ideals of Ω.

**Theorem 3.8.** Let $\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma \in \Omega\}$ be a QN-*Q*-I of *Q*-algebra Ω. Then, the F-Sets $\{(\gamma, T_{\tilde{A}}(\gamma)): \gamma\in\Omega\}$, $\{(\gamma, C_{\tilde{A}}(\gamma)): \gamma\in\Omega\}$, $\{(\gamma, 1\text{-}U_{\tilde{A}}(\gamma)): \gamma\in\Omega\}$ and $\{(\gamma, 1\text{-}F_{\tilde{A}}(\gamma)): \gamma\in\Omega\}$ are F-*Q*-Is of Ω.

**Proof.** Assume that $\tilde{A} = \{(\gamma, T_{\tilde{A}}(\gamma), C_{\tilde{A}}(\gamma), U_{\tilde{A}}(\gamma), F_{\tilde{A}}(\gamma)) : \gamma \in \Omega\}$ be a QN-*Q*-I of a *Q*-algebra Ω. Therefore,

(i) $T_{\tilde{A}}(0) \geq T_{\tilde{A}}(\gamma)$ & $T_{\tilde{A}}(\gamma * \delta) \geq \min\{T_{\tilde{A}}((\gamma * \alpha) * \delta), T_{\tilde{A}}(\alpha)\}$, $\forall\gamma, \delta, \alpha\in\Omega$;

(ii) $C_{\tilde{A}}(0) \geq C_{\tilde{A}}(\gamma)$ & $C_{\tilde{A}}(\gamma * \delta) \geq \min\{C_{\tilde{A}}((\gamma * \alpha) * \delta), C_{\tilde{A}}(\alpha)\}$, $\forall\gamma, \delta, \alpha\in\Omega$;

(iii) $U_{\tilde{A}}(0) \leq U_{\tilde{A}}(\gamma)$ & $U_{\tilde{A}}(\gamma * \delta) \leq \max\{U_{\tilde{A}}((\gamma * \alpha) * \delta), U_{\tilde{A}}(\alpha)\}$, $\forall\gamma, \delta, \alpha\in\Omega$;

(iv) $F_{\tilde{A}}(0) \leq F_{\tilde{A}}(\gamma)$ & $F_{\tilde{A}}(\gamma * \delta) \leq \max\{F_{\tilde{A}}((\gamma * \alpha) * \delta), F_{\tilde{A}}(\alpha)\}$, $\forall\gamma, \delta, \alpha\in\Omega$.

Clearly, $T_{\tilde{A}}(0) \geq T_{\tilde{A}}(\gamma)$ & $T_{\tilde{A}}(\gamma * \delta) \geq \min\{T_{\tilde{A}}((\gamma * \alpha) * \delta), T_{\tilde{A}}(\alpha)\}$, $\forall\gamma, \delta, \alpha\in\Omega$. Therefore, the F-Set $\{(\gamma, T_{\tilde{A}}(\gamma)): \gamma\in\Omega\}$ is a F-*Q*-I of Ω.

Clearly, $C_{\tilde{A}}(0) \geq C_{\tilde{A}}(\gamma)$ & $C_{\tilde{A}}(\gamma * \delta) \geq \min\{C_{\tilde{A}}((\gamma * \alpha) * \delta), C_{\tilde{A}}(\alpha)\}$, $\forall\gamma, \delta, \alpha\in\Omega$. Therefore, the F-Set $\{(\gamma, C_{\tilde{A}}(\gamma)): \gamma\in\Omega\}$ is a F-*Q*-I of Ω.

Now, $U_{\tilde{A}}(\gamma * \delta) \leq \max\{U_{\tilde{A}}((\gamma * \alpha) * \delta), U_{\tilde{A}}(\alpha)\}$, $\forall\gamma, \delta, \alpha\in\Omega$

$\Rightarrow 1\text{-}U_{\tilde{A}}(\gamma) \geq \min\{1\text{-}U_{\tilde{A}}((\gamma * \alpha) * \delta), 1\text{-}U_{\tilde{A}}(\alpha)\}, \forall\gamma, \delta, \alpha\in\Omega$

and $U_{\tilde{A}}(0) \leq U_{\tilde{A}}(\gamma), \forall\gamma\in\Omega$

$\Rightarrow 1\text{-}U_{\tilde{A}}(0) \geq 1\text{-}U_{\tilde{A}}(\gamma), \forall\gamma\in\Omega$.

Hence, the F-Set $\{(\gamma, 1\text{-}U_{\tilde{A}}(\gamma)): \gamma\in\Omega\}$ is a F-*Q*-I of Ω.

Now, $F_{\tilde{A}}(\gamma * \delta) \leq \max\{F_{\tilde{A}}((\gamma * \alpha) * \delta), F_{\tilde{A}}(\alpha)\}$, $\forall\gamma, \delta, \alpha\in\Omega$

$\Rightarrow 1\text{-}F_{\tilde{A}}(\gamma) \geq \min\{1\text{-}F_{\tilde{A}}((\gamma * \alpha) * \delta), 1\text{-}F_{\tilde{A}}(\alpha)\}, \forall\gamma, \delta, \alpha\in\Omega$

and $F_{\tilde{A}}(0) \leq F_{\tilde{A}}(\gamma), \forall\gamma\in\Omega$

$\Rightarrow 1\text{-}F_{\tilde{A}}(0) \geq 1\text{-}F_{\tilde{A}}(\gamma), \forall\gamma\in\Omega$.

Hence, the F-Set $\{(\gamma, 1\text{-}F_{\tilde{A}}(\gamma)): \gamma\in\Omega\}$ is a F-*Q*-I of Ω.

**5.**

## Conclusions

In this article, we have grounded the concept of QN-*Q*-I of QN-*Q*-A as a generalization of IF-*Q*-I of IF-*Q*-A. By defining QN-*Q*-I, QN-*Q*-sub-algebra, we have formulated several results on QN-*Q*-A under QNS environment. In the future, based on the notion of QN-*Q*-A and QN-*Q*-I many new investigations can be carried out by the researchers those are working on N-Set, QNS, and their extensions.

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