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Application of a limit function of negative hypergeometric distribution in option pricing

Aplicación de una función límite de distribución hipergeométrica negativa en la valoración de opciones

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Resumen

Este trabajo presenta la función límite de una distribución hipergeométrica negativa que se aplica en la valoración de opciones utilizando la ecuación de riqueza y algunas herramientas de martingala. Este artículo presenta un modelo de tiempo discreto simple en comparación con otro modelo existente. Este trabajo concluye que el límite de la hipergeometría negativa se puede asociar con términos financieros que se pueden usar para evaluar los valores de las opciones (sin dividendos), lo que da el mismo valor numérico que el modelo CRR.

Palabras clave: Distribución negativa hipergeométrica, ecuación de riqueza y opción.

Abstract

This work introduces limit function of a negative hyper geometric distribution which is applied in option pricing using wealth equation and some martingale tools. This paper presents a simple discrete time model in compares with another existing model. This work concludes that limit of negative hyper geometric can be associated with financial terms that can be used to evaluate option values (non dividend) which gives the same numerical with CRR model.

Keywords: Negative hyper geometric distribution, wealth equation and option

1. Introduction

This paper focuses on a negative hyper-geometric distribution discussed in [5] which is given as of the form

$$nh_{R,S,r}(x) = \frac{\binom{r+x-1}{x}\binom{R-r+S-x}{S-x}}{\binom{R+S}{S}} \qquad x = 0,1 \ldots\ldots S \quad (1.0)$$

where $R, S \in \mathbb{N}$ and $r \in \{1, \ldots . R\}$.The mean and Variance of X are given as $\frac{rS}{R+1}$ and $\frac{rS(R+S+1)(R-r+1)}{(R+1)^2(R+2)}$ respectively. If the terms and condition on the parameters of negative hyper geometric distribution are satisfy then the limit function of the distribution can be association with finance term to determine European option pricing. To enable the purchasing or selling of an option, we would like to be able to determine its value at any point in time. This paper aims to answer the question of option pricing under a simplified framework using the limit function of a negative hyper geometric distribution.

For example John Cox et al [3] gave a recursive procedure for finding the value of call with any number of periods to go with general formula for any n:

$$C = \left[\sum_{j=0}^{n}\left(\frac{n!}{j!(n-j)!}\right)p^j(1-p)^{n-j}max[u^j d^{n-j}S - K]\right]/r^n \qquad (1.1)$$

Now the proposed model of the form

$$X_0 = \frac{1}{(1+r)^T}\sum_k^T\binom{T}{k}\frac{r^k(R+r-1)^{T-k}}{(R+1)^T}max\left[u^k d^{n-k}S_{(0)} - K\right]$$

where $k = 0,1 \ldots T$ (1.2)

In this paper, we make a characterization taking into account results presented in ([1], [2],[4]). For more notions related to the topic presents in this paper, we refer the reader to [9].

2. Method

Let X be the negative hygrometric random variable with the parameters $R, T,$ and r. Its probability function is given as of the form

$$P_x(X) = \frac{\binom{r+k-1}{k}\binom{R-r+T-k}{T-k}}{\binom{R+T}{T}} \quad x = 0,1 \ldots\ldots T$$

If $P_x(X) = \frac{\binom{r+k-1}{k}\binom{R-r+T-k}{S-k}}{\binom{R+T}{T}}$

$$= \binom{T}{k}\frac{\frac{(r\pm k-1)!}{(r-1)!}\frac{(R-r+T-k)!}{(R-r)!}}{\frac{(R+T)!}{(T)!}} = \binom{T}{k}\frac{(r+k-1)!}{(r-1)!}\frac{(R-r+T-k)!}{(R-r)!} \times \frac{T!}{(R+T)!}$$

$$\binom{T}{k}\frac{(R+k-1)(r+k-2)\ldots\ldots r \times (R-r+K-k)(R-r+T-k-1)\ldots(R-r+1)}{(R+T)(R+T-1)\ldots\ldots(R+1)}$$

$$= \binom{T}{k}\frac{(R+1)\left[\frac{(r\pm 1)r+k-2}{R+1\quad R+1}\ldots\ldots\frac{r}{R+1}\right]\times(R+1)\left[\frac{(R-r+T-k)(R-r+T-k-1)}{R+1\qquad R+1}\ldots\frac{R-r+1}{R+1}\right]}{(R+1)\left[\frac{(R+T)(R+T-1)}{R+1\quad R+1}\ldots\ldots\ldots 1\right]}$$

$$= \binom{T}{k}\frac{(R+1)^k\left[\frac{r}{(R+1)}\ldots\frac{(r+k-2)(r+k-1)}{(R+1)\quad(R+1)}\right]\times(R+1)^{T-k}\left[\frac{(R-r+1)}{(R+1)}\ldots\ldots\frac{(R-r+T-k-1)(R-r+T-k)}{(R+1)\qquad(R+1)}\right]}{(R+1)^T\left[1\ldots\frac{(R+T)}{(R+1)}\right]}$$

If $R, r \to \infty$ while $\frac{r}{R+1}$ and 1-$\frac{r}{R+1}$ remain constant.

$$P_x(X) \to \left(\frac{r}{R+1}\right)^k\left(\frac{R-r+1}{R+1}\right)^{S-T}$$

$$P_x(X) \to \left(\frac{r}{R+1}\right)^k\left(\frac{R-r+1}{R+1}\right)^{S-T} = \frac{r^k(R+r-1)^{T-k}}{(R+1)^T} \qquad (2.0)$$

Let the wealth equation defined on subset $\omega = \{H, T\}$

$$X_{k+1} = \Delta_k S_{k+1} + (1+r)(X_k - \Delta_k S_k) \qquad (2.1)$$

where Δ_k is the share of stock , for $(X_k - \Delta_k S_k)$ invested in the money market .

where $k = 0,1, \ldots N$

Solving (2.1)

$$X_{k+1} = \Delta_k S_{k+1} + (1+r)(X_k - \Delta_k S_k)$$

$$(1+r)X_k + \Delta_k(S_{k+1} - (1+r)S_k) = X_{k+1}$$

$$X_k + \Delta_k\left[\frac{1}{(r+1)}S_{k+1}(H) - S_k\right] = \frac{X_{k+1}}{(r+1)}$$

$$X_k + \Delta_k\left[\frac{1}{(r+1)}S_{k+1}(H) - S_k\right] = \frac{X_{k+1}(H)}{(1+r)} \qquad (2.2)$$

$$X_k + \Delta_k\left[\frac{1}{(r+1)}S_{k+1}(T) - S_k\right] = \frac{X_{k+1}(T)}{(r+1)} \qquad (2.3)$$

Let $uS_k = S_{k+1}(H)$ and $S_{k+1}(T) = dS_k$

From equations 2.2 and 2.3, it follows that

$$\Delta_k = \frac{[X_{k+1}(H)X_{k+1} - X_{k+1}(T)]}{[u-d]S_k} \qquad (2.4)$$

2.1 Assumptions

1. The initial value of the stock is $S(0)$ that is stock price at $T = 0$
2. At the end of the period , the price is the go up with a factor u or go down with a factor d, where $u = \frac{S_1(H)}{S_0} > 1$ and $d = \frac{S_1(T)}{S_0} < 1$ at $T = 1$
3. At the end of the period the price is either $uS(0)$ with neutral probabilities $\frac{r}{R+1}$ or $d\,S(0)$ with probability $\frac{R-r+1}{R+1}$.
4. The movement can also be traced from a view point of tossing a coin ,which result to a head and tail .if it result to a head at a time one we have $S_{k+1}(H) = uS_n$, if it result to a tail at a time one ,we have $S_{k+1}(T) = dS_n$
5. Assume $d < u$, if $d > u$ relabel ; if $d = u$ then S_n is not a random .
6. Let $R = (1+r)$ and $0 < 1+r < u$, then no arbitrage

Lemma 2.1 : If $0 < d < 1+r < u$, then no arbitrage where d, r and u are followed at was mentioned in assumptions 2.1 .

Proof:

Let consider$1+r \geq u > d$, and portfolio Z:$\left[x = -\frac{1}{S(0)}, y = \frac{1}{B(0)}\right]$where $B(0)$ denotes the price of the bond at $t = 0$.

Let $V_Z(0)$ and $V_Z(1)$ respectively denote the value of the portfolio Z at $t = 0$ and $t = 1$.

$$V_z(0) = xS(0) + yB(0)$$

$$= -\frac{1}{S(0)}S(0) + \frac{1}{B(0)}B(0) = 0$$

and

$$V_z(0) = xS(1) + yB(1)$$

$$= \begin{cases} -\frac{1}{S(0)}uS(0) + \frac{1}{B(0)}(1+r)B(0) & with\ probability\ > 1 \\ -\frac{1}{S(0)}dS(0) + \frac{1}{B(0)}(1+r)B(0) & with\ probability\ < 1 \end{cases}$$

$$= \begin{cases}(1+r) - u \geq 0 \\ (1+r) - d > 0\end{cases} \qquad (2.5\ )$$

(2.5) violates the no arbitrage

Lets $u > d \geq (1+r)$

Constructing a portfolio Q:$\left[x = \frac{1}{S(0)}, y = -\frac{1}{B(0)}\right]$

$$V_Q(0) = xS(0) - yB(0) = 0$$

$$= \begin{cases} \frac{1}{S(0)}uS(0) - \frac{1}{B(0)}(1+r)B(0) & with\ probability\ > 1 \\ \frac{1}{S(0)}dS(0) - \frac{1}{B(0)}(1+r)B(0) & with\ probability\ < 1 \end{cases}$$

$$= \begin{cases} u - (1+r) > 0 \\ d - (1+r) \geq 0 \end{cases} \qquad (2.6)$$

(2.6) clearly violates no arbitrage thus $0 < d < 1+r < u$ is justified.

Lemma 2.2: Let $\frac{r}{R+1} = \frac{1+r-d}{u-d}$ and $1 - \frac{r}{R+1} = \frac{u-1-r}{u-d}$ exist for $0 < \frac{r}{R+1} < 1 - \frac{r}{R+1}$, the following holds

i. $\frac{r}{R+1} + 1 - \frac{r}{R+1} = \frac{1+r-d}{u-d} + \frac{u-1-r}{u-d} = 1$

ii. $\left(\frac{r}{R+1}\right) u + \left(1 - \frac{r}{R+1}\right) d \times \frac{1}{(1+r)} = 1$

iii. $\sum_{i=1}^{3} \frac{r}{R+1_i} = 1 \ \forall \ i = 1,2 \ldots 3$

Proof:

I and ii are followed. Then , we proves iii. Let $\sum_{i=1}^{3} \frac{r}{R+1_i} = 1 \ \forall \ i = 1,2,3 \ldots$

Defining $\frac{r}{R+1_1} = \left(\frac{r}{R+1}\right)^2, \frac{r}{R+1_2} = 2\frac{r}{R+1}\left(1 - \frac{r}{R+1}\right)$ and $\frac{r}{R+1_3} = \left(1 - \frac{r}{R+1}\right)^2$

$\sum_{i=1}^{3} \frac{r}{r+b_i} = \sum_{l=1}^{3} \left[\left(\frac{r}{R+1}\right)^2 + 2\frac{r}{R+1}\left(1 - \frac{r}{R+1}\right) + \left(1 - \frac{r}{R+1}\right)^2\right]$.

$= \left[\left(\frac{r}{R+1}\right)^2 - \left(\frac{r}{R+1}\right)^2 + 1\right] = 1$

Lemma 2.3 let $S_{k+1}(H) = uS_k$ and $S_{k+1}(T) = dS_k$ then $\frac{S_k}{(1+r)^k} = \mathbb{E}\left[\frac{S_{k+1}}{(1+r)^{k+1}}\right]$ where $k = 0,1 \ldots N$.

Proof :

If $\left(\frac{r}{R+1}\right) u + \left(1 - \frac{r}{R+1}\right) d \times \frac{1}{(1+r)} = 1$

Multiplying both side by S_k

$\frac{1}{(1+r)}\left[\left(\frac{r}{R+1}\right) uS_k + \left(1 - \frac{r}{R+1}\right) dS_k\right] = S_k$

Multiplying both by $\frac{1}{(1+r)^k}$

$\frac{S_k}{(1+r)^k} = \mathbb{E}\left[\frac{S_{k+1}}{(1+r)^{k+1}}\right]$.

Lemma 2.4: If $\Delta_k = \frac{[X_{k+1}(H) - X_{k+1}(T)]}{S_{k+1}(H) - S_{k+1}(T)}$ then the discounted wealth process under risk neutral measure.

$\frac{X_k}{(1+r)^k}$ $k = 0,1 \ldots N$ is martingale.

Proof:

Multiplying (2.2) by $\frac{r}{R+1}$ and (2.3) $\frac{R-r+1}{R+1}$

$$X_k + \Delta_k\left[\left(\frac{r}{R+1}\right)\frac{1}{R+1} S_{k+1}(H) - \left(\frac{1}{R+1}\right) S_k\right] = \left(\frac{1}{1+r}\right)\left(\frac{r}{R+1}\right) X_{k+1}(H) \quad (2.7)$$

$$X_k + \Delta_k\left[\left(\frac{R-r+1}{R+1}\right)\frac{1}{R+1} S_{k+1}(T) - \left(\frac{R-r+1}{R+1}\right) S_k\right] = \left(\frac{1}{1+r}\right)\left(\frac{R-r+1}{R+1}\right) X_{k+1}(T) \quad (2.8)$$

Adding (2.7) and (2.8)

$$X_k + \Delta_k\left[\frac{1}{1+r}\left(\frac{1}{R+1} S_{k+1}(H)\right) + \left(\frac{R-r+1}{R+1}\right) S_k\right] - S_k = \frac{1}{1+r}\left[\left(\frac{r}{R+1}\right) X_{k+1}(H) + \left(\frac{R-r+1}{R+1}\right) X_{k+1}(T)\right]$$

By lemma 2.3

$$X_k = \frac{1}{1+r}\left[\left(\frac{r}{R+1}\right) X_{k+1}(H) + \left(\frac{R-r+1}{R+1}\right) X_{k+1}(T)\right] \quad (2.9)$$

Multiplying both side by $\frac{1}{(1+r)^k}$

$$\frac{X_k}{(1+r)^k} = \frac{1}{(1+r)^{k+1}}\left[\left(\frac{r}{R+1}\right) X_{k+1}(H) + \left(\frac{R-r+1}{R+1}\right) X_{k+1}(T)\right]$$

$$\frac{X_k}{(1+r)^k} = \mathbb{E}\frac{[X_{k+1}]}{(1+r)^{k+1}} .$$

Set $k = 0$ implies time zero (2.9) reduces to

$$X_0 = \frac{1}{1+r}\left[\left(\frac{r}{R+1}\right) X_1(H) + \left(\frac{R-r+1}{R+1}\right) X_1(T)\right] \quad (2.10)$$

Theorem 1.1

Given wealth equation (2.1) defined on the subset $\omega = \{HH, HT, TH, TT\}$ with a unfair probability and with $\Delta_k = \frac{[X_{k+1}(HH) - X_{k+1}(HT)]}{S_{k+1}(HH) - S_{k+1}(HT)}$ a discounted wealth process under risk neutral measure. Then

$$X_k = \frac{1}{(1+r)^k}\sum_k^T \binom{T}{k}\frac{r^k (R+r-1)^{T-k}}{(R+1)^T} max[u^k d^{k-x} S_n - K] \quad (2.11)$$

Proof:

$X_{k+1} = \Delta_k S_{k+1} + (1+r)(X_k - \Delta_k S_k)$

Defining it on the subset $\omega = \{HH, HT, TH, TT\}$ of tossing a coins.

$$X_{k+1} = \begin{cases} X_{k+1}(HH) = \Delta_k S_{k+1}(HH) + (1+r)\big(X_k(H) - \Delta_k S_k(H)\big) \\ X_{k+1}(HT) = \Delta_k S_{k+1}(HT) + (1+r)\big(X_k(H) - \Delta_k S_k(H)\big) \\ X_{k+1}(TT) = \Delta_k S_{k+1}(TT) + (1+r)\big(X_k(H) - \Delta_k S_k(H)\big) \end{cases}$$

Solving (2.12) and (2.13)

$$\Delta_k = \frac{X_{k+1}(HH) - X_{k+1}(HT)}{S_{k+1}(HH) - S_{k+1}(HT)} \quad (2.15)$$

Solving (2.13) and (2.14)

$$\Delta_k = \frac{X_{k+1}(HT) - X_{k+1}(TT)}{S_{k+1}(HT) - S_{k+1}(TT)} \quad (2.16)$$

Then

$X_{k+1}(HH) = \Delta_k S_{k+1}(HH) + (1+r)\big(X_k(H) - \Delta_k S_k(H)\big)$

$(1+r)X_k(H) + \Delta_k S_{k+1}(HH) - (1+r)\Delta_k S_k(H) = X_{k+1}(HH)$

$(1+r)X_k(H) + \Delta_k[S_{k+1}(HH) - (1+r)S_k(H)] = X_{k+1}(HH)$

By (2.15)

$(1+r)X_k(H) + \frac{X_{k+1}(HH) - X_{k+1}(HT)}{S_{k+1}(HH) - S_{k+1}(HT)}[S_{k+1}(HH) - (1+r)S_k(H)] = X_{k+1}(HH)$

Choose $X_{k+1}(HH) = C_{uu}, X_k(H) = C_u$, $X_{k+1}(HT) = C_{ud}$, and $S_{k+1}(HH) = uS_n(H), S_{k+1}(HT) = dS_n(H)$

$(1+r)C_u + \frac{C_{uu} - C_{ud}\,[uS_n(H) - (1+r)S_n(H)]}{uS_n(H) - dS_n(H)} = C_{uu}$

$(1+r)C_u + \frac{C_{uu} - C_{ud}[u-(1+r)]S_n(H)}{(u-d)S_n(H)} = C_{uu}$

$(1+r)C_u + \frac{C_{uu}[u-(1+r)]}{u-d} - \frac{C_{ud}[u-(1+r)]}{u-d}$

By lemma 2.2

$(1+r)C_u + \frac{R+r-1}{R+1}C_{uu} - \frac{r}{R+1}C_{ud} = C_{uu}$

$(1+r)C_u = C_{uu} - \frac{R+r-1}{R+1}C_{uu} + \frac{R+r-1}{R+1}C_{ud}$

$(1+r)C_u = \left(1-\frac{R+r-1}{R+1}\right)C_{uu} + \left(\frac{R+r-1}{R+1}\right)C_{ud}$

$(1+r)C_u = \left(\frac{r}{R+1}\right)C_{uu} + \left(\frac{R+r-1}{R+1}\right)C_{ud}$

$C_u = \frac{1}{1+r}\left[\left(\frac{r}{R+1}\right)C_{uu} + \left(\frac{R+r-1}{R+1}\right)C_{ud}\right]$

Set $k=1$ implies at a time one

$X_1(H) = \frac{1}{(r+1)}\left[\frac{r}{R+1}X_2(HH) + \frac{R+r-1}{R+1}X_2(HT)\right]$

Thus

$X_1(T) = \frac{1}{(r+1)}\left[\frac{r}{R+1}X_2(HT) + \frac{R+r-1}{R+1}X_2(TT)\right]$

From lemma 2.4

$X_0 = \frac{1}{1+r}\left[\left(\frac{r}{R+1}\right)X_1(H) + \left(\frac{R-r+1}{R+1}\right)X_1(T)\right]$

$X_0 = \frac{1}{1+r}\left[\left(\frac{r}{R+1}\right)\left(\frac{1}{(r+1)}\left[\left(\frac{r}{R+1}\right)X_2(HH) + \left(\frac{R+r-1}{R+1}\right)X_2(HT)\right]\right) + \left(\frac{R-r+1}{R+1}\right)\left(\frac{1}{(r+1)}\left[\frac{r}{R+1}X_2(HT) + \frac{R+r-1}{R+1}X_2(TT)\right]\right)\right]$

$X_0 = \frac{1}{1+r}\left[\frac{1}{1+r}\left(\left(\frac{r}{R+1}\right)^2 X_2(HH) + 2\left(\frac{r}{R+1}\right)\left(\frac{R+r-1}{R+1}\right)X_2(HT) + \left(\frac{R-r+1}{R+1}\right)^2 X_2(TT)\right)\right]$

$X_0 = \frac{1}{(1+r)^2}\left(\frac{r}{R+1} + \frac{R+r-1}{R+1}\right)^2 max[u^x d^{n-x} S_0 - K, 0]$

For time two we have

$X_0 = \frac{1}{(1+r)^2}\sum_{k=0}^{2}\binom{2}{k}\frac{r^k(R+r-1)^{2-k}}{(R+1)^2} \quad max[u^x d^{n-x}S_0 - K, 0]$

For general purpose

$$X_0 = \frac{1}{(1+r)^T}\sum_{k=0}^{T}\binom{T}{k}\frac{r^k(R+r-1)^{T-k}}{(R+1)^T} \quad max[u^k d^{n-k}S_0 - K, 0] \qquad (2.16)$$

3. Numerical Results

The following examples have been given to illustrate the application of limit function hyper geometric distribution.

Example 3.1: Given that $S_0 = 100$, $K = 100$, $u = 1.2$, $d = 0.8$ $r = 10\%$ and $T = 2$.

$X_0 = \frac{1}{(1+r)^T}\sum_{k=0}^{T}\binom{T}{k}\frac{r^k(R+r-1)^{T-k}}{(R+1)^T} max[u^k d^{n-k}S_0 - K, 0]$

$X_0 = \frac{1}{1.1^2}\left[\frac{2!}{2!0!}\frac{(3)^2\times(3)^0}{(4)^2}\times 44\right] = \frac{1}{1.21}[1\times 0.562\times 1\times 44] = \$20.4$

CRR Model

$C = \left[\sum_{j=0}^{n}\left(\frac{n!}{j!(n-j)!}\right)p^j(1-p)^{n-j}Max[0, u^j d^{n-j}S - K]/r^n\right]$

$C = \frac{1}{1.1^2}\left[\frac{2!}{0!2!}\left(\frac{25}{100}\right)^2\left(\frac{75}{100}\right)^0\times 0 + \frac{2!}{1!1!}\left(\frac{25}{100}\right)^1\left(\frac{75}{100}\right)^1\times 0 + \frac{2!}{2!0!}\left(\frac{25}{100}\right)^0\left(\frac{75}{100}\right)^2\times 44\right] = \$20.45$

Example 3.2: Let $S_0 = 100$, $K = 100, r = 7\%$, $T = 3$, $u = 1.1$ and $d = 0.9$

$. X_0 = \frac{1}{(1+r)^T}\sum_{k=0}^{T}\binom{T}{k}\frac{r^k(R+r-1)^{T-k}}{(R+1)^T} \quad max[u^k d^{n-k}S_0 - K, 0]$

$X_0 = \frac{1}{(1.07)^3}\left[\frac{3!}{0!3!}\frac{(85)^0\times(15)^3}{(100)^3}\times 0 + \frac{3!}{1!2!}\frac{(85)^1\times(15)^2}{(100)^3}\times 0 + \frac{3!}{2!1!}\frac{(85)^2\times(15)^1}{(100)^3}\times 8.90 + \frac{3!}{3!0!}\frac{(85)^3\times(15)^0}{(100)^3}\times 33.10\right]$

= \$18.96

CRR Model

$C = \left[\sum_{j=0}^{n}\left(\frac{n!}{j!(n-j)!}\right)p^j(1-p)^{n-j}Max[0, u^j d^{n-j}S - K]/r^n\right]$

$C = \frac{1}{(1.07)^3}\left[\frac{3!}{0!3!}\frac{(85)^0\times(15)^3}{(100)^3}\times 0 + \frac{3!}{1!2!}\frac{(85)^1\times(15)^2}{(100)^3}\times 0 + \frac{3!}{2!1!}\frac{(85)^2\times(15)^1}{(100)^3}\times 8.90 + \frac{3!}{3!0!}\frac{(85)^3\times(15)^0}{(100)^3}\times 33.10\right] = \$18.96$

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4. Conclusion

Cox ad Ross [3] gave and option pricing model which is famously called CRR model for option pricing. In compares this work formulates a model using negative hypergeometric distribution when $R, r\to\infty$, which concludes an alternative numerical procedure which is both simpler and for many purpose of computationally more efficient. On the other hand, for future works, results obtained in this paper can be applied or extended in Neutrosophic theory by applying Neutrosophic random variables (see [6,7,8]).

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