BISTUA Rev. FCB, Volumen 20 (1) (2022) 63-69. Pamplona-Colombia.

BISTUA
Revista de la Facultad de Ciencias Básicas
ISSN: 0120-4211 - ISSN-e: 2711-3027

Degree, Order and Size of Single-Valued Quadripartitioned Neutrosophic Graphs

Grado, orden y tamaño de grafos neutrosóficos cuadriparticionado

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***Recibido**: Mayo, 2022.* ***Aceptado**: Junio, 2022.* ***Publicado**: Junio, 2022*

Resumen
En este artículo se introduce y estudia el concepto de gráfico neutrosófico cuadriparticionado de un solo valor (SVQN-grafo) extendiendo la idea de la teoría del gráfico neutrosófico de un solo valor (SVN-grafo). Además, se define la noción de grado, orden y tamaño de los SVQN-grafos. Además, proporcionamos algunos ejemplos ilustrativos para justificar los resultados.

Palabras clave: SVNS; SVN-grafo; SVQN-grafo; SVQN-conjunto.

Abstract
In this article an attempt is made to introduce and study the concept of single-valued quadripartitioned neutrosophic graph (SVQN-graph) by extending the idea of single-valued neutrosophic graph (SVN-graph) theory. Besides, we introduce the notion of degree, order and size of SVQN-graphs. Further, we furnish few illustrative examples to justify the results.

Keywords: SVNS; SVN-graph; SVQN-graph; SVQN-set.

1. Introduction

Graph theory is generally used as a tool to deal with the combinatorial problems in number theory, geometry, topology, algebra, etc. In the year 1986, Biggs et al. [7] presented the concept of graph theory. Bollobas [8] introduced the idea of modern graph theory. Later on, Gonzalez et al. [15] metric locating dominating sets of graphs. To deal with the situation having uncertainty, Sunitha and Mathew [23] presented a survey of fuzzy graph in 2013. Recently, Pal et al. [21] further studied F-graph theory. Shannon and Atanassov [24] developed intuitionistic F-graph based on intuitionistic FS (in short IFS). Afterwards, Dhavudh and Srinivasan [13] introduced the idea of intuitionistic fuzzy graph of second type. Intuitionistic F-graph have been further studied by Nagoor Gani and Shajitha Begum [20], Sahoo and Pal [25]. Later on, Akram and Akmal [1] studied the intuitionistic fuzzy graph structure in the year 2017. The notion of intuitionistic fuzzy graph from the point of view of α, β, and $(\alpha;\beta)$-levels was introduced by Shannon and Atanassov [26]. Afterwards, Parvathi and Karunambigai [22] further studied intuitionistic fuzzy graphs. To deal with inconsistency and indeterminacy Prof. F. Smarandache developed the notion of neutrosophic set (NS) in the year 1998. Broumi et al. [9] introduced the idea of single valued neutrosophic graph and studied their degree, order, and size. Later on Akram and Sitara [2], Akram [3] further studied the structure of single valued neutrosophic graphs. Afterwards, Akram [4] introduced the notion of single valued neutrosophic planar graphs. In the year 2017, Akram and Shahzadi [5] presented the idea of neutrosophic soft graph and give a real life application of neutrosophic soft graph. Akram et al. [6] grounded the concept of neutrosophic soft rough graph and proposed a MADM strategy. Thereafter, Sayed et al. [27] introduced the notion of rough neutrosophic digraphs with a real life application. The concept of neutrosophic vague graph was introduced and studied by Hussain et al. [16]. In the year 2020, Hussain et al. [17] presented the notion of neutrosophic vague line graph. The idea of homomorphism and isomorphism in strong neutrosophic graphs was grounded by Mullai et al. [19] in the year 2020. The notion of interval-valued neutrosophic graph was introduced and studied by Singh [28]. In the year 2020, Mukherjee and Das [18] introduced the neutrosophic bipolar vague soft set and proposed an application of it. In the year 2016, Chatterjee et al. [11] grounded the notion of quadripartitioned neutrosophic set and proposed a multi criteria decision making strategy based on the similarity measure. Later on, Das et al. [14] applied the concept of topology on quadripartitioned neutrosophic sets and introduced the notion of quadripartitioned neutrosophic topological space.

The rest of this article has been organized into four sections:

Section-2 is on preliminaries and definitions those are relevant for developing the main results of this article. In section-3, the notion of degree, order and size of SVQN-graphs have been procured and some properties of those have been investigated. In section-4, we conclude the paper, and state some future scope of research in this direction.

2. Some relevant results

Definition 2.1.[11] Suppose that $\dot{G}$ be a fixed set. Then, a single valued quadripartitioned neutrosophic set (SVQN-set) H over $\dot{G}$ is defined by:

$H = \{(ω, Ţ_H(ω), Ç_H(ω), Ų_H(ω), Ņ_H(ω)) : ω∈\dot{G}\}$.

Here, $Ţ_H$, $Ç_H$, $Ų_H$, $Ņ_H$ are the truth, contradiction, unknown and falsity membership functions respectively from $\dot{G}$ to [0, 1]. So, $0 ≤ Ţ_H(ω) + Ç_H(ω) + Ų_H(ω) + Ņ_H(ω) ≤ 4$, for each $ω∈\dot{G}$.

Definition 2.2.[11] Suppose that $A=\{(ω, Ţ_A(ω), Ç_A(ω), Ų_A(ω), Ņ_A(ω)) : ω∈\dot{G}\}$ and $B=\{(ω, Ţ_B(ω), Ç_B(ω), Ų_B(ω), Ņ_B(ω)) : ω∈\dot{G}\}$ be two SVQN-sets over $\dot{G}$. Then, A is said to be a subset of B (i.e., $A⊆B$) if and only if $Ţ_A(ω) ≤ Ţ_B(ω)$, $Ç_A(ω) ≤ Ç_B(ω)$, $Ų_A(ω) ≥ Ų_B(ω)$, $Ņ_a(ω) ≥ Ņ_B(ω)$, for each $ω ∈ \dot{G}$.

Definition 2.3.[11] Suppose that $A=\{(ω, Ţ_A(ω), Ç_A(ω), Ų_A(ω), Ņ_A(ω)) : ω∈\dot{G}\}$ and $B=\{(ω, Ţ_B(ω), Ç_B(ω), Ų_B(ω), Ņ_B(ω)) : ω∈\dot{G}\}$ be two SVQN-sets over $\dot{G}$. Then, the union of A and B is defined by

$A∪B = \{(ω, \max\{Ţ_A(ω),Ţ_B(ω)\}, \max\{Ç_A(ω),Ç_B(ω)\}, \min\{Ų_A(ω),Ų_B(ω)\}, \min\{Ņ_A(ω),Ņ_A(ω)\}) : ω∈\dot{G}\}$.

Definition 2.4.[11] Suppose that $A=\{(ω, Ţ_A(ω), Ç_A(ω), Ų_A(ω), Ņ_A(ω)) : ω∈\dot{G}\}$ be a SVQN-set over $\dot{G}$. Then, the complement of A is defined by $A^c=\{(ω, Ņ_A(ω), Ų_A(ω), Ç_A(ω), Ţ_A(ω)) : ω∈\dot{G}\}$.

Definition 2.5.[11] Suppose that $A=\{(ω, Ţ_A(ω), Ç_A(ω), Ų_A(ω), Ņ_A(ω)) : ω∈\dot{G}\}$ and $B=\{(ω, Ţ_B(ω), Ç_B(ω), Ų_B(ω), Ņ_B(ω)) : ω∈\dot{G}\}$ be two SVQN-sets over $\dot{G}$. Then, the intersection of A and B is defined by $A∩B=\{(ω, \min\{Ţ_A(ω),Ţ_B(ω)\}, \min\{Ç_A(ω), Ç_B(ω)\}, \max\{Ų_A(ω), Ų_B(ω)\}, \max\{Ņ_A(ω), Ņ_B(ω)\}) : ω∈\dot{G}\}$.

Definition 2.6.[10] Suppose that Θ be a fixed set of n vertex. Assume that Δ be the set of edges between the vertices. Then, $\hat{G}=(Ψ, Ω)$ is called a single-valued neutrosophic graph (SVN-graph),

where (i) $T_Ψ$, $I_Ψ$, $F_Ψ$: Θ→[0, 1] denotes the truth, indeterminacy and false membership functions of a vertex $ω_i∈Θ$ respectively such that $0 ≤ T_Ψ(ω_i) + I_Ψ(ω_i) + F_Ψ(ω_i) ≤ 3$ $∀ω_i∈Θ$, (i=1, 2, …., n).

(ii) $T_Ω$, $I_Ω$, $F_Ω$: $Δ⊆Θ×Θ→[0, 1]$ defined by

$T_Ω(ω_i, ω_j) ≤ \min\{T_Ω(ω_i), T_Ω(ω_j)\}$,

$I_Ω(ω_i, ω_j) ≥ \max\{I_Ω(ω_i), I_Ω(ω_j)\}$,

$F_Ω(ω_i, ω_j) ≥ \max\{F_Ω(ω_i), F_Ω(ω_j)\}$,

denotes the truth, indeterminacy and false membership functions of the edge $(ω_i, ω_j)∈Δ$, respectively such that $0 ≤ T_Ω(ω_i, ω_j) + I_Ω(ω_i, ω_j) + F_Ω(ω_i, ω_j) ≤ 3$ ($∀(ω_i, ω_j)∈Δ$, i=1, 2, …., n).

Here, Ψ is called the SVQN vertex set of Θ and Ω is said to be the SVQN edge set of Δ respectively.

3. Single-valued quadripartitioned neutrosophic graph

Definition 3.1. Suppose that $Θ = \{ω_i: i=1, 2, …, n\}$ be a fixed set of vertices and $Δ = \{(ω_i, ω_j): i, j=1, 2, …, n\}$ be the set of edges between the vertices of Θ. An SVQN-graph of $\hat{G}^*=(Θ, Δ)$ is defined by $\hat{G}=(Ψ, Ω)$, where (i) $Ţ_Ψ$:Θ→[0, 1], $Ç_Ψ$:Θ→[0, 1], $Ų_Ψ$:Θ→[0, 1] and $Ņ_Ψ$:Θ→[0, 1] denotes the truth, contradiction, unknown and false membership functions of the vertices $ω_i∈Θ$ respectively such that $0 ≤ Ţ_Ψ(ω_i) + Ç_Ψ(ω_i) + Ų_Ψ(ω_i) + Ņ_Ψ(ω_i) ≤ 4$, $∀ω_i ∈Θ$ (i=1, 2, …, n);

(ii) $Ţ_Ω$:$Δ⊆Θ×Θ$→[0, 1], $Ç_Ω$:$Δ⊆Θ×Θ$→[0, 1], $Ų_Ω$:$Δ⊆Θ×Θ$→[0, 1] and $Ņ_Ω$:$Δ⊆Θ×Θ$→[0, 1] defined by $Ţ_Ω(ω_i, ω_j) ≤ \min\{Ţ_Ψ(ω_i), Ţ_Ψ(ω_j)\}$,

$Ç_Ω(ω_i, ω_j) ≤ \min\{Ç_Ψ(ω_i), Ç_Ψ(ω_j)\}$,

$Ų_Ω(ω_i, ω_j) ≥ \max\{Ų_Ψ(ω_i), Ų_Ψ(ω_j)\}$,

and $Ņ_Ω(ω_i, ω_j) ≥ \max\{Ņ_Ψ(ω_i), Ņ_Ψ(ω_j)\}$,

indicates the truth, contradiction, unknown and false-membership functions from $Δ⊆Θ×Θ$ to [0, 1], respectively such that $0 ≤ Ţ_Ψ(ω_i) + Ç_Ψ(ω_i) + Ų_Ψ(ω_i) + Ņ_Ψ(ω_i) ≤ 4$, $∀(ω_i, ω_j)∈Δ$ (i, j = 1, 2, …., n). Here, Ψ is the SVQN vertex set of Θ and Ω is the SVQN edge set of Δ respectively. Therefore, $\hat{G}=(Ψ, Ω)$ is an SVQN-graph of $\hat{G}^*=(Θ, Δ)$ if $Ţ_Ω(ω_i, ω_j) ≤ \min\{Ţ_Ψ(ω_i), Ţ_Ψ(ω_j)\}$; $Ç_Ψ(ω_i, ω_j) ≤ \min\{Ç_Ψ(ω_i), Ç_Ψ(ω_j)\}$; $Ų_Ψ(ω_i, ω_j) ≥ \max\{Ų_Ψ(ω_i), Ų_Ψ(ω_j)\}$; and $Ņ_Ψ(ω_i, ω_j) ≥ \max\{ Ņ_Ψ(ω_i), Ņ_Ψ(ω)\}$.Clearly, both Ψ and Ω are the SVQN-set over Θ and E respectively.

Example 3.1. Assume that $\hat{G} = (Θ, Δ)$ is a graph, where $Θ = \{ω_1, ω_2, ω_3, ω_4\}$ and $Δ = \{(ω_1, ω_2), (ω_2, ω_3), (ω_3, ω_4), (ω_4, ω_1)\}$. Suppose that Ψ is an SVQN vertex set of Θ and Ω is an SVQN edge set of Δ defined by the Table 1.

Table 1: Tabular representation of Example 3.1

	$ω_1$	$ω_2$	$ω_5$	$(ω_1, ω_2)$	$(ω_1, ω_5)$
$Ţ_{Ψ'}$	0.1	0.2	0.0	0.0	0.0
$C_{Ψ'}$	0.2	0.1	0.5	0.1	0.1
$U_{Ψ'}$	0.6	0.6	0.6	0.8	0.8
$F_{Ψ'}$	0.8	0.6	0.7	0.9	1.0

The graph of Example 3.1 is presented in Fig. 1

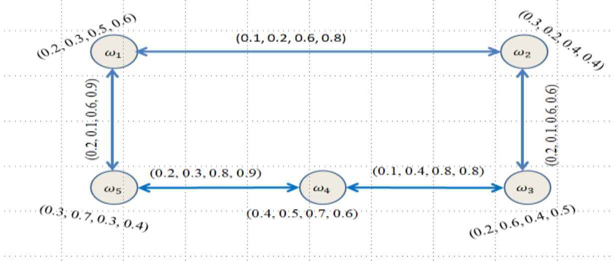

Figure 1: SVQN-graph

Therefore, $\hat{G} = (Ψ, Ω)$ is an SVQN-graph of $\hat{G}^* = (Θ, Δ)$.

Remark 3.1. Assume that $\hat{G} = (Ψ, Ω)$ is an SVQN-graph. Then, the edge $(ω_i, ω_j)$ is said to be incident at $ω_i$ and $ω_j$.

Definition 3.2. Suppose that $\hat{G} = (Ψ, Ω)$ be an SVQN-graph. Then,

(i) $(ω_i, Ţ_Ψ(ω_i), Ç_Ψ(ω_i), Ų_Ψ(ω_i), Ņ_Ψ(ω_i))$ is called a single valued quadripartitioned neutrosophic vertex (in short SVQN-vertex).

(ii) $((ω_i, ω_j), Ţ_Ω((ω_i, ω_j)), Ç_Ω((ω_i, ω_j)), Ų_Ω((ω_i, ω_j)), Ņ_Ω((ω_i, ω_j)))$ is called an SVQN edge (in short SVQN-edge).

Definition 3.3. Suppose that $\hat{G} = (Ψ, Ω)$ be an SVQN-graph of $\hat{G}^* = (Θ, Δ)$. Then, $H = (Ψ', Ω')$ is called an SVQN sub-graph (in short SVQN-sub-graph) of $\hat{G} = (Ψ, Ω)$ if $H = (Ψ', Ω')$ is also an SVQN-graph of $\hat{G}^* = (Θ, Δ)$ such that

(i) $Ψ' \subseteq Ψ$ i.e., $Ţ'_Ψ(ω_i) \le Ţ_Ψ(ω_i)$, $Ç'_Ψ(ω_i) \le Ç_Ψ(ω_i)$, $Ų'_Ψ(ω_i) \ge Ų_Ψ(ω_i)$, and $Ņ'_Ψ(ω_i) \ge Ņ_Ψ(ω_i)$, for all $ω_i \in Θ$;

(ii) $Ω' \subseteq Ω$ i.e., $Ţ'_Ω((ω_i, ω_j)) \le Ţ_Ω((ω_i, ω_j))$, $Ç'_Ω((ω_i, ω_j)) \le Ç_Ω((ω_i, ω_j))$, $Ų'_Ω((ω_i, ω_j)) \ge Ų_Ω((ω_i, ω_j))$, and $Ņ'_Ω((ω_i, ω_j)) \ge Ņ_Ω((ω_i, ω_j))$, $\forall (ω_i, ω_j) \in Δ$.

Example 3.2. Assume that $\hat{G} = (Ψ, Ω)$ be an SVQN-graph of $\hat{G}^* = (Θ, Δ)$ as shown in Example 1. Then, $H = (Ψ', Ω')$, where $Ψ' = \{ω_1, ω_2, ω_5\}$, $Ω' = \{(ω_1, ω_2), (ω_1, ω_5)\}$ defined by the Table 2:

Table 2. Tabular representation of Example 3.2

	$ω_1$	$ω_2$	$ω_3$	$ω_4$	$ω_5$	$(ω_1, ω_2)$	$(ω_2, ω_3)$	$(ω_3, ω_4)$	$(ω_4, ω_5)$	$(ω_5, ω_1)$
$Ţ_Ψ$	0.2	0.3	0.2	0.4	0.3	0.1	0.2	0.1	0.2	0.2
$Ç_Ψ$	0.3	0.2	0.6	0.5	0.7	0.2	0.1	0.4	0.3	0.1
$Ų_Ψ$	0.5	0.4	0.4	0.7	0.3	0.6	0.6	0.8	0.8	0.6
$Ņ_Ψ$	0.6	0.4	0.5	0.6	0.4	0.8	0.6	0.8	0.9	0.9

The graph of Example 3.2 is presented in Fig. 2

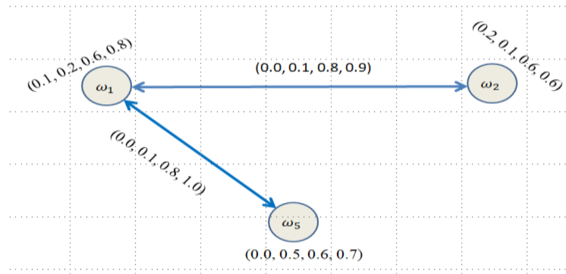

Figure 2: SVQN-sub-graph

Here, $H = (Ψ', Ω')$ is an SVQN-sub-graph of $\hat{G} = (Ψ, Ω)$.

Definition 3.4. Suppose that $\hat{G} = (Ψ, Ω)$ be an SVQN-graph of $\hat{G}^* = (Θ, Δ)$. Then, the complement of $\hat{G} = (Ψ, Ω)$ is an SVQN-graph $\bar{\hat{G}}$ of $\hat{G}^* = (Θ, Δ)$, where

(i) $\bar{Ţ}_Ψ(ω_i) = Ţ_Ψ(ω_i)$, $\bar{Ç}_Ψ(ω_i) = Ç_Ψ(ω_i)$, $\bar{Ų}_Ψ(ω_i) = Ų_Ψ(ω_i)$, $\bar{Ņ}_Ψ(ω_i) = Ņ_Ψ(ω_i)$, $\forall ω_i \in Θ$;

(ii) $\bar{Ţ}_Ω(ω_i, ω_j) = \min\{Ţ_Ω(ω_i), Ţ_Ψ(ω_j)\} - Ţ_Ω(ω_i, ω_j)$, $\bar{Ç}_Ω(ω_i, ω_j) = \min\{Ç_Ω(ω_i), Ç_Ψ(ω_j)\} - Ç_Ω(ω_i, ω_j)$,

$\bar{Ų}_Ω(ω_i, ω_j) = \max\{Ų_Ψ(ω_i), Ų_Ψ(ω_j)\} - Ų_Ω(ω_i, ω_j)$, and $\bar{Ņ}_Ω(ω_i, ω_j) = \max\{Ņ_Ψ(ω_i), Ņ_Ψ(ω_j)\} - Ņ_Ω(ω_i, ω_j)$, $\forall (ω_i, ω_j) \in Δ$.

Definition 3.5. Suppose that $\hat{G} = (Ψ, Ω)$ be an SVQN-graph. Then, the vertices $ω_i$ and $ω_j$ are called adjacent in $\hat{G} = (Ψ, Ω)$ if and only if $Ţ_Ω(ω_i, ω_j) = \min\{Ţ_Ψ(ω_i), Ţ_Ψ(ω_j)\}$, $Ç_Ω(ω_i, ω_j) = \min\{Ç_Ψ(ω_i), Ç_Ψ(ω_j)\}$, $Ų_Ω(ω_i, ω_j) = \max\{Ų_Ψ(ω_i), Ų_Ψ(ω_j)\}$ and $Ņ_Ω(ω_i, ω_j) = \max\{Ņ_Ψ(ω_i), Ņ_Ψ(ω_j)\}$.

Example 3.3. Assume that $\hat{G} = (Ψ, Ω)$ be an SVPN-graph, which is defined in following Table 3.

Table 3. Tabular representation of Example 3.3

	$ω_1$	$ω_2$	$ω_3$	$(ω_1, ω_2)$	$(ω_2, ω_3)$	$(ω_3, ω_1)$
$Ţ_Ψ$	0.1	0.3	0.2	0.1	0.2	0.1
$Ç_Ψ$	0.2	0.6	0.4	0.2	0.4	0.2
$Ų_Ψ$	0.9	0.6	0.5	0.9	0.6	0.9
$Ņ_Ψ$	0.8	0.6	0.6	0.8	0.6	0.8

The graph of Example 3.3 is presented in Fig. 3

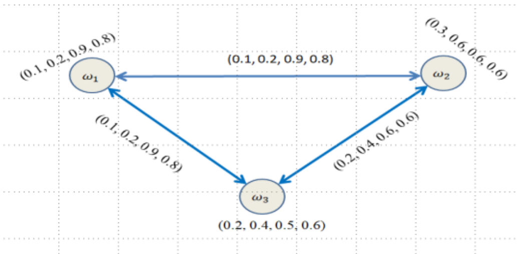

Figure 3: SVQN-graph of Adjacent Vertices

Here, the vertices ω_1 and ω_2 are adjacent in the SVQN-graph $\hat{G} = (\Psi, \Omega)$. Similarly, the vertices ω_3 and ω_1 are adjacent in the SVQN-graph $\hat{G} = (\Psi, \Omega)$. But the vertices ω_2 and ω_3 are not adjacent in the SVQN-graph $\hat{G} = (\Psi, \Omega)$.

Definition 3.6. In an SVQN-graph $\hat{G} = (\Psi, \Omega)$, a vertex $\omega_j \in \Theta$ is called an isolated vertex if there exists no edge incident at ω_j.

Example 3.4. Suppose that $\hat{G} = (\Psi, \Omega)$ be an SVQN-graph, which is defined in Table 4.

Table 4. Tabular representation of Example 3.4

	ω1	**ω2**	**ω3**	**ω4**	**(ω1, ω2)**	**(ω2, ω4)**	**(ω4, ω1)**
Ţ$_{\Psi}$	0.4	0.3	0.6	0.2	0.2	0.1	0.2
Ç$_{\Psi}$	0.3	0.4	0.8	0.9	0.2	0.3	0.2
Ų$_{\Psi}$	0.5	0.4	0.6	0.6	0.6	0.8	0.9
Ņ$_{\Psi}$	0.5	0.6	0.6	0.9	0.4	0.5	1.0

The graph of Example 3.4 is represented in Fig. 4

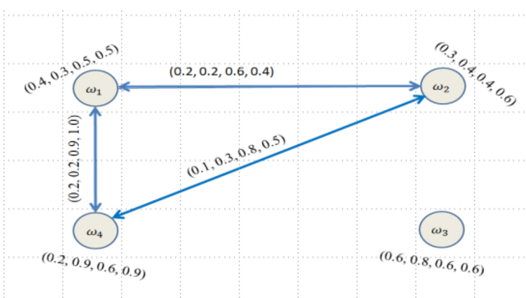

Figure 4: SVQN-graph with Isolated vertex

In the above SVQN-graph $\hat{G} = (\Psi, \Omega)$, the vertex ω_3 is an isolated vertex.

Definition 3.7. Suppose that $\hat{G} = (\Psi, \Omega)$ is an SVQN-graph. Assume that ω_0 and ω_n be two vertices in $\hat{G} = (\Psi, \Omega)$. Then, an SVQN path $P(\omega_0, \omega_n)$ in an SVQN-graph $\hat{G} = (\Psi, \Omega)$ is a sequence of distinct vertices $\omega_0, \omega_1, \omega_2, \omega_3, \ldots, \omega_n$ such that Ţ$_{\Omega}(\omega_{i-1}, \omega_i) > 0$, Ç$_{\Omega}(\omega_{i-1}, \omega_i) > 0$, Ų$_{\Omega}(\omega_{i-1}, \omega_i) > 0$ and Ņ$_{\Omega}(\omega_{i-1}, \omega_i) > 0$, where $0 \leq i \leq n$. Here, $n\ (\geq 1)$ is called the length of the path $P(\omega_0, \omega_n)$. The consecutive pairs (ω_{i-1}, ω_i) $(0 \leq i \leq n)$ are called the edges of the path $P(\omega_0, \omega_n)$. The path $P(\omega_0, \omega_n)$ is called a cycle if $\omega_0 = \omega_n$, where $n \geq 3$.

Definition 3.8. Suppose that $\hat{G} = (\Psi, \Omega)$ be an SVQN-graph. Then, $\hat{G} = (\Psi, \Omega)$ is said to be an SVQN connected graph (in short SVQN-C-graph) if there exists at least one SVQN-path between two vertices.

Definition 3.9. Assume that $\hat{G} = (\Psi, \Omega)$ be an SVQN-graph. Then, a vertex having exactly one edge incident on it is called a pendent vertex. If a vertex is not a pendent vertex, then it is called a non-pendent vertex.

Remark 3.2. Suppose that $\hat{G} = (\Psi, \Omega)$ be an SVQN-graph.

(i) If an edge is incident with a pendent vertex, then the edge is said to be a pendent edge. Otherwise, it is called a non-pendent edge.

(ii) If a vertex is adjacent to a pendent vertex, then the vertex is said to be a support of that pendent edge.

Example 3.5. Let $\hat{G} = (\Psi, \Omega)$ be an SVQN-graph, which is defined by Table 9 and Table 10.

Table 5: Tabular representation of Example 3.5

	ω1	**ω2**	**ω3**	**ω4**	**(ω1, ω2)**	**(ω2, ω3)**	**(ω3, ω4)**
Ţ$_{\Psi}$	0.2	0.4	0.5	0.6	0.1	0.3	0.4
Ç$_{\Psi}$	0.5	0.4	0.3	0.2	0.3	0.2	0.1
Ų$_{\Psi}$	0.6	0.8	0.9	0.7	0.9	0.9	1.0
Ņ$_{\Psi}$	0.8	0.7	0.8	0.7	0.9	0.9	0.9

The graph of Example 3.5 is represented in Fig. 5

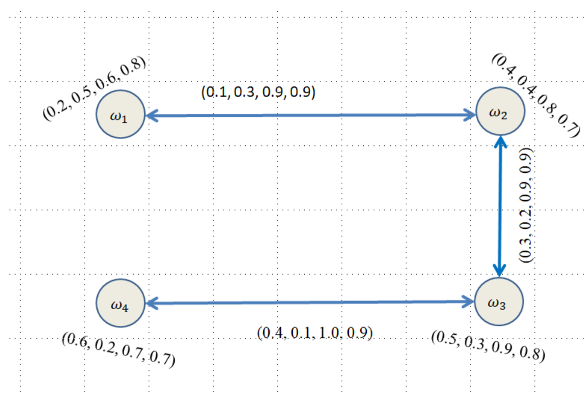

Figure 5: SVQN-graph with Pendent Vertex

In the above SVQN-graph $\hat{G} = (\Psi, \Omega)$, the vertices ω_1 and ω_4 are the pendent vertices. But the vertices ω_2 and ω_3 are the non-pendent vertices. Similarly, the edges (ω_1, ω_2) and (ω_3, ω_4) are the pendent edges. But the edge (ω_2, ω_3) is a non-pendent edge. The vertex ω_3 is support of the pendent edge (ω_3, ω_4). But ω_2 is not the support of the pendent edge (ω_1, ω_2).

Definition 3.10. A SVQN-graph $\hat{G} = (\Psi, \Omega)$ of $\hat{G}^* = (\Theta,\Delta)$ is said to be a complete SVQN-graph if

$Ţ_\Omega(\omega_i, \omega_j) = \min \{Ţ_\Psi(\omega_i), Ţ_\Psi(\omega_j)\}$;

$Ç_\Omega(\omega_i, \omega_j) = \min \{Ç_\Psi(\omega_i), Ç_\Psi(\omega_j)\}$;

$Ų_\Omega(\omega_i, \omega_j) = \max \{Ų_\Psi(\omega_i), Ų_\Psi(\omega_j)\}$;

and $Ņ_\Omega(\omega_i, \omega_j) = \max \{Ņ_\Psi(\omega_i), Ņ_\Psi(\omega_j)\}, \forall \omega_i, \omega_j \in \Theta$.

Example 3.6. Assume that $\hat{G}^* = (\Theta, \Delta)$ is a graph, where $\Theta = \{\omega_1, \omega_2, \omega_3\}$ and $\Delta = \{(\omega_1, \omega_2), (\omega_2, \omega_3), (\omega_3, \omega_1)\}$. Suppose that $\hat{G} = (\Psi, \Omega)$ be an SVQN-graph defined by Table 6.

Table 6. Tabular representation of Example 3.6

	ω_1	ω_2	ω_3	ω_4	ω_5	(ω_1, ω_2)	(ω_1, ω_3)	(ω_4, ω_2)	(ω_4, ω_3)	(ω_5, ω_2)	(ω_5, ω_3)
$Ţ_\Psi$	0.5	0.4	0.2	0.6	0.5	0.2	0.1	0.3	0.1	0.3	0.1
$Ç_\Psi$	0.4	0.4	0.2	0.4	0.5	0.1	0.1	0.3	0.1	0.3	0.2
$Ų_\Psi$	0.1	0.5	0.4	0.5	0.4	0.6	0.5	0.7	0.6	0.5	0.5
$Ņ_\Psi$	0.4	0.5	0.5	0.4	0.8	0.6	0.5	0.6	0.6	0.9	1.0

The graph of Example 3.6 is represented in Fig. 6

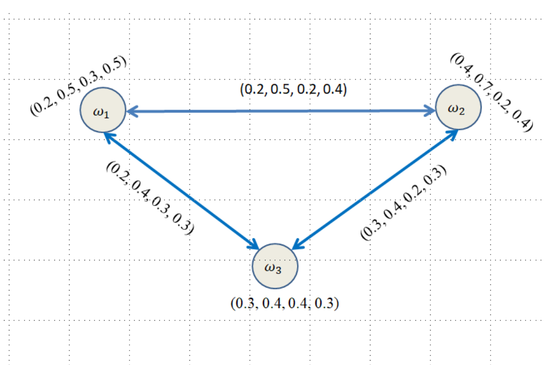

Figure 6: Complete SVQN-graph

Here, the above graph is a complete SVQN-graph.

Table 7. Tabular representation of Example 3.7

	ω_1	ω_2	ω_3	(ω_1, ω_2)	(ω_2, ω_3)	(ω_3, ω_1)
$Ţ_\Psi$	0.2	0.4	0.3	0.2	0.3	0.2
$Ç_\Psi$	0.5	0.7	0.4	0.5	0.4	0.4
$Ų_\Psi$	0.3	0.2	0.4	0.2	0.2	0.3
$Ņ_\Psi$	0.5	0.4	0.3	0.4	0.3	0.3

Definition 3.11. A SVQN-graph $\hat{G} = (\Psi, \Omega)$ of $\hat{G}^* = (\Theta, \Delta)$ is called a bipartite SVQN-graph if the graph $\hat{G}^* = (\Theta,\Delta)$ is a bipartite graph.

Example 3.7. Assume that $\hat{G}^* = (\Theta, \Delta)$ be a graph, where $\Theta = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}$ and $\Delta = \{(\omega_1, \omega_2), (\omega_2, \omega_3), (\omega_3, \omega_1)\}$. Suppose that $\hat{G} = (\Psi, \Omega)$ be an SVQN-graph defined by Table 7.

The representation of the graph of Example 3.7 is presented in Fig. 7

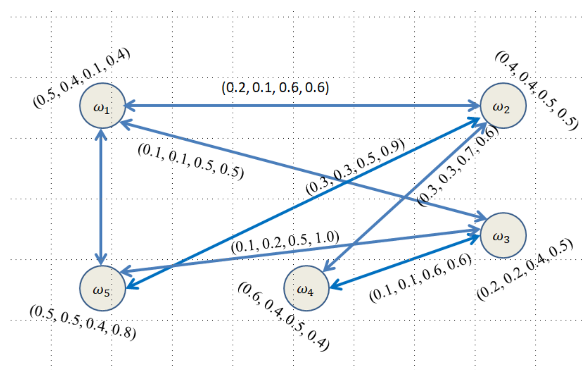

Figure 7: Bipartite SVQN-graph

Here, the crisp graph $\hat{G}^* = (\Theta, \Delta)$ is a bipartite graph and $\hat{G} = (\Psi, \Omega)$ is a SVQN-graph of $\hat{G}^* = (\Theta, \Delta)$. Hence, $\hat{G} = (\Psi, \Omega)$ is a bipartite SVQN-graph.

Definition 3.12. Suppose that $\hat{G} = (\Psi, \Omega)$ be an SVQN-graph. Then, the degree of the vertex ω is defined by

$d(\omega) = (d_Ţ(\omega), d_Ç(\omega), d_Ų(\omega), d_Ņ(\omega))$,

where $d_Ţ(\omega)$ = degree of the truth-membership vertex = sum of the truth-membership of all edges those are incident on the vertex $\omega = \sum_{u \neq \omega} Ţ_\Omega (u, \omega)$;

Table 8. Tabular representation of Example 3.8

	ω1	ω2	ω3	ω4	(ω1, ω2)	(ω2, ω3)	(ω3, ω4)	(ω4, ω1)
$Ţ_Ψ$	0.3	0.5	0.5	0.8	0.3	0.2	0.3	0.2
$Ç_Ψ$	0.4	0.3	0.5	0.6	0.3	0.2	0.3	0.2
$Ų_Ψ$	0.5	0.4	0.4	0.5	0.5	0.6	0.5	0.6
$Ņ_Ψ$	0.6	0.5	0.3	0.8	0.6	0.9	0.6	0.9

$d_Ç(ω)$ = degree of the contradiction-membership vertex = sum of the contradiction-membership of all edges those are incident on the vertex ω = $\sum_{u≠ω} Ç_Ω(u, k)$;

$d_Ų(ω)$ = degree of the unknown-membership vertex = sum of the unknown-membership of all edges those are incident on the vertex ω = $\sum_{u≠ω} Ų_Ω(u, ω)$;

$d_Ņ(ω)$ = degree of the falsity-membership vertex = sum of the false-membership of all edges those are incident on the vertex ω = $\sum_{u≠ω} Ņ_Ω(u, ω)$.

Example 3.8. Assume that Ĝ = (Ψ, Ω) be an SVQN-graph of Ĝ* = (Θ, Δ) defined by Table-8.

Table 9: Tabular representation of Example 3.9

	ω1	ω2	ω3	ω4	(ω1, ω2)	(ω2, ω3)	(ω3, ω4)	(ω4, ω1)
$Ţ_Ψ$	0.2	0.4	0.4	0.5	0.2	0.3	0.2	0.1
$Ç_Ψ$	0.4	0.5	0.4	0.6	0.3	0.2	0.3	0.4
$Ų_Ψ$	0.4	0.4	0.2	0.9	0.8	0.5	1.0	0.9
$Ņ_Ψ$	0.3	0.3	0.2	0.4	0.5	0.4	0.5	0.8

The representation of the graph of Example 3.8 is shown in Fig. 8

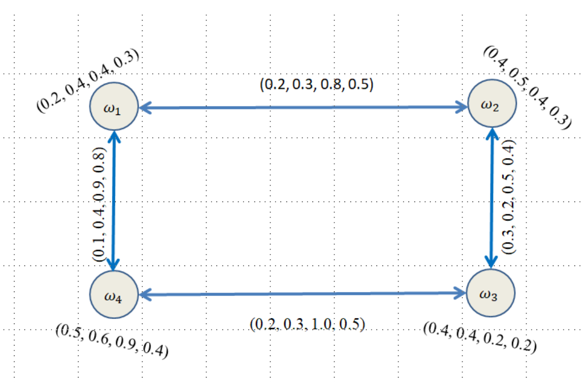

Figure 8: SVQN-graph for Example 8

Then, $d(ω_1)$ = (0.3, 0.7, 1.7, 1.3), $d(ω_2)$ = (0.5, 0.5, 1.3, 0.9), $d(ω_3)$ = (0.5, 0.5, 1.5, 0.9), and $d(ω_4)$ = (0.3, 0.7, 1.9, 1.3).

Definition 3.13. Suppose that Ĝ = (Ψ, Ω) is an SVQN-graph of Ĝ* = (Θ, Δ). Then, Ĝ = (Ψ, Ω) is called a constant SVQN-graph if degree of each vertices is same i.e., d(ω) = ($d_Ţ(ω)$, $d_Ç(ω)$, $d_Ų(ω)$, $d_Ņ(ω)$), ∀ω∈Θ.

Example 3.9. Assume that Ĝ = (Ψ, Ω) be an SVQN-graph, which is defined by Table 9

The representation of the graph for Example 3.9 is shown in Fig. 9

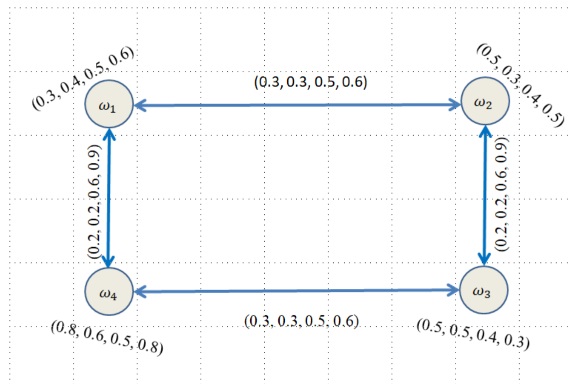

Figure 9: SVQN-graph for Example 9

In the above SVQN-graph Ĝ = (Ψ, Ω), the degree of the vertices $ω_1$, $ω_2$, $ω_3$, and $ω_4$ are $d(ω_1)$ = (0.5, 0.5, 1.1, 1.5), $d(ω_2)$ = (0.5, 0.5, 1.1, 1.5), $d(ω_3)$ = (0.5, 0.5, 1.1, 1.5) and $d(ω_4)$ = (0.5, 0.5, 1.1, 1.5). Hence, Ĝ = (Ψ, Ω) is a constant SVQN-graph.

Definition 3.14. Assume that Ĝ = (Ψ, Ω) be a SVQN-graph. Then, the order of Ĝ = (Ψ, Ω), denoted by O(Ĝ) is defined by

O(Ĝ) = ($O_Ţ$(Ĝ), $O_Ç$(Ĝ), $O_Ų$(Ĝ), $O_Ņ$(Ĝ)),

where $O_Ţ$(Ĝ) = $\sum_{ω∈V} Ţ_Ψ$ denotes the Ţ-order of Ĝ=(Ψ, Ω);

$O_Ç$(Ĝ) = $\sum_{ω∈V} Ç_Ψ$ denotes the Ç-order of Ĝ=(Ψ, Ω)

$O_Ų$(Ĝ) = $\sum_{ω∈V} Ų_Ψ$ denotes the Ų-order of Ĝ=(Ψ, Ω);

O_F(Ĝ) = $\sum_{ω∈V} Ņ_Ψ$ denotes the Ņ-order of Ĝ=(Ψ, Ω).

Example 3.10. Assume that Ĝ = (Ψ, Ω) be an SVQN-graph of Ĝ* = (Θ, Δ) as shown in Example 8. Then, the order of the SVQN-graph Ĝ = (Ψ, Ω) is O(Ĝ) = (1.5, 1.9, 1.9, 1.2).

Definition 3.15. Suppose that Ĝ = (Ψ, Ω) is an SVQN-graph. Then, the size of Ĝ = (Ψ, Ω), denoted by S(Ĝ) is defined by

S(Ĝ) = ($S_Ţ$(Ĝ), $S_Ç$(Ĝ), $S_Ų$(Ĝ), $S_Ņ$(Ĝ)),

where $S_Ţ$(Ĝ) = $\sum_{u≠ω} Ţ_Ω(u, k)$ denotes the Ţ-size of Ĝ = (Ψ, Ω);

S_C(Ĝ) = $\sum_{u≠ω} Ç_Ω(u, k)$ denotes the Ç-size of Ĝ = (Ψ, Ω);

$S_Ų$(Ĝ) = $\sum_{u≠ω} Ų_Ω(u, k)$ denotes the Ų-size of Ĝ = (Ψ, Ω);

$S_Ņ$(Ĝ) = $\sum_{u≠ω} Ņ_Ω(u, k)$ denotes the Ņ-size of Ĝ = (Ψ, Ω).

Example 3.11. Assume that Ĝ = (Ψ, Ω) be an SVQN-graph of Ĝ* = (Θ, Δ) as shown in Example 8. Then, the size of the SVQN-graph Ĝ = (Ψ, Ω) is S(Ĝ) = (0.8, 1.2, 3.2, 2.2).

4. Conclusions

This article presents the notion of degree, order, and size of SVQN-graphs. Further, few examples have been furnished to justify the definitions and results established. It is hoped that, the notion presented in this paper will open up new avenues of research on SVQN-graph for its application in real problems in the current single valued quadripartitioned neutrosophic area.

Conflict of Interest: The authors declare that they have no conflict of interest.

Authors Contribution: All the authors have equal contribution for the preparation of this article.

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